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# Designing Efficient Algorithms for Logistics Management: Optimizing Time-Constrained Vehicle Routing

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**Abstract**

**Background**: City logistics is a critical component of urban economic development, as it optimizes supply chains, enhances customer satisfaction through reliable deliveries, and minimizes environmental impacts in densely populated areas. This field addresses various challenges, including traffic congestion, environmental concerns, noise pollution, and the crucial need for timely deliveries. Routing and scheduling are central to logistics operations, with modern software integrating time windows to meet precise scheduling demands driven by detailed customer requirements and operational efficiencies. Furthermore, advanced vehicle routing models now effectively simulate real-world factors such as traffic congestion, stochastic travel times, and dynamic product demands.

**Purpose:** This paper aims to develop an algorithm that addresses the routing decisions. Our approach extends to the time dimension, considering travel times and customer service times within predefined time windows.

**Study design/methodology/approach**: The proposed algorithm is structured to execute in iterative phases, aiming to optimize key logistical objectives. In order to generate competitive solutions, we seek to minimize the number of vehicles utilized and overall travel costs. The evaluation of solution space was conducted via Simulated Annealing.

**Findings/conclusions:** The performance of the proposed algorithm, evaluated using the Gehring and Homberger benchmark instances for 200 customers, demonstrates its effectiveness. The algorithm successfully meets the target number of vehicles required, and the associated travel costs are on average within 1% of the best solutions reported in the relevant literature.

**Limitations/future research:** Given the ongoing need for timely solutions from decision-makers, future research endeavors will focus on enhancing the computational efficiency of the algorithm. Additionally, incorporating more time-related features, such as stochastic travel times, could further improve the algorithm's real-time applicability.

**Keywords**
City logistics, Vehicle routing, Scheduling, Time windows, Simulated annealing

## Introduction

City logistics plays an important role in sustaining urban economies by optimizing supply chain efficiency, improving customer satisfaction through dependable delivery services, and mitigating the environmental repercussions of freight transportation in densely populated areas. This field encompasses the management of challenges like traffic congestion, spatial constraints on roads, environmental impacts, noise pollution, and the critical requirement for punctual deliveries. Among the crucial models considered in applications and research is multi-echelon vehicle routing.

The Vehicle Routing Problem (VRP) representing a critical logistical challenge is extensively studied in Operations Research. Originating in 1959 with Dantzig and Ramser's formulation of the Truck Dispatching Problem, it aimed to minimize travel distances for a homogeneous fleet delivering to gas stations from a central depot. This seminal work laid the groundwork for subsequent research into optimizing delivery routes. In 1964, Clarke and Wright expanded on this concept by formulating a generalized linear optimization problem. This advancement enabled the efficient distribution of goods from a central depot to geographically dispersed consumers using vehicles with varying capacities. Known today as the VRP, this problem has become fundamental in logistics and transportation research, guiding strategies to minimize costs and improve efficiency in delivering goods to diverse locations.

Since its inception, the VRP has evolved to handle complexities like varying vehicle capacities, strict time constraints, multiple depots, and other practical challenges in logistics. Route optimization is crucial in transportation, logistics, and supply chain management, serving sectors such as public transit, postal services, food delivery, cash logistics, and waste management. Designing efficient transport routes involves considering factors like vehicle characteristics, capacities, travel costs, service time windows, and other variables. These elements determine the feasibility and effectiveness of tailored route plans for specific operational needs.

Reducing the number of vehicles and travel distance is key to cutting vehicle costs, lowering product prices, and reducing greenhouse gas emissions. Various real-world routing problems aim to minimize total logistics expenses while maintaining high service levels. Transportation costs are a significant part of product expenses, split into fixed (like driver wages and vehicle maintenance) and variable costs (like fuel expenses, which depend on route duration and specific factors). Efficient vehicle routing not only improves economic efficiency but also promotes environmental sustainability by minimizing emissions and operational impacts.

Lenstra and Rinnooy Kan proved in 1981 that the VRP is NP-hard, indicating that exact algorithms are efficient only for small-scale instances. Therefore, heuristic and metaheuristic algorithms are more suitable for solving real-world problems of significant size. Modern approaches in VRP algorithms emphasize both exact and approximate solutions. Exact algorithms are constrained to smaller problems due to the problem's complexity. In contrast, approximate algorithms offer quick but not necessarily optimal solutions, and are a primary focus of algorithmic research. Metaheuristics like simulated annealing, tabu search, ant colony optimization, genetic algorithms, and memetic algorithms are widely used for tackling VRP and related optimization challenges.

Today's vehicle routing models have advanced significantly, integrating complex real-world factors such as stochastic travel times to simulate traffic congestion, time windows for precise product collection and delivery scheduling, and varying dynamic demands for products. Each of these enhancements introduces substantial intricacy into the optimization process. In contemporary vehicle routing models, routes are typically planned for specific time periods, often daily. This can translate into different operational scenarios: routes may be recalculated iteratively with updated data inputs, or a single route plan may repeat over an extended timeframe. The Vehicle Routing Problem with Time Windows (VRPTW) is a critical variant where each customer requires service within a designated time slot. For instance, some customers may specify deliveries only between 9 AM and 10 AM. The integration of time windows into modern logistics software has gained significant traction over the past decades, driven by increasingly detailed customer needs and operational efficiencies. As logistics challenges continue to evolve, new variations and enhancements to routing algorithms and software solutions will undoubtedly remain at the forefront of research and development in city logistics and broader transportation management.

Real-world logistics rely on precise algorithms within routing software to optimize delivery routes amidst complex constraints. For instance, in fast parcel delivery, algorithms ensure efficient routes, uniform delivery schedules, and minimal customer-driver interactions, boosting service quality and operational efficiency. Similar optimizations benefit diverse sectors like newspaper delivery, school bus routing, waste management, postal services, and security patrols, addressing challenges such as variable travel times and fluctuating demand. In today's competitive landscape, advanced routing systems are critical for meeting customer expectations and maintaining a strategic edge in logistics.

The rest of the paper is organized as follows. Section 1 contains the overview of literature on VRP and related time variations. In Section 2 we give a problem description and mathematical model. Section 3 is devoted to presentation of the algorithm, followed by the results in Section 4.

## 1. Related literature

Literature on VRP and related variations is very rich. We will emphasize several papers related to time-constrained versions. For a more detailed overview, one can see for instance Bräysy and Gendreau (2005a and 2005b) or Konstantakopoulos et al. (2022).

Solomon's (1985) paper addresses vehicle routing and scheduling problems with time window constraints by developing and analyzing tour-building algorithms for the VRPTW. He extends existing VRP heuristics to integrate both distance and time dimensions into the heuristic process, resulting in more adaptable methods capable of handling time window constraints. Initially, he adapted Clarke and Wright (1964) savings heuristic for the VRPTW to enhance route optimization effectiveness. Additionally, Solomon explores sequential tour-building algorithms and insertion heuristics that initialize routes based on various criteria. These methods iteratively insert customers into partial routes, optimizing tour efficiency within specified time constraints. Li and Wang's study (2025) addresses the capacitated vehicle routing problem in the context of omnichannel retailing, accounting for multiple types of time windows and products simultaneously.

Exact methods often exhibit considerable inefficiencies, sometimes taking days or more to find even moderately satisfactory, let alone optimal, solutions for relatively small problem instances. A comprehensive analysis of well-known exact algorithms for VRP can be found in Laporte (1992). Fisher et al. (1997) introduce an algorithm that solves the VRPTW optimally, formulating the problem as a K-tree with a degree of 2K at the depot. Kohl and Madsen (1997) present a shortest path approach with side constraints, followed by Lagrange relaxation. This method relaxes constraints to ensure each customer is served exactly once. Desrosiers et al. (1984) pioneer the use of the column generation approach for solving VRPTW, and a more effective version incorporating valid inequalities achieves optimality in Desrosiers et al. (1992). The dynamic programming approach for VRPTW is first presented by Kolen et al. (1987), while Christofides and Beasley (1984) utilize the dynamic programming paradigm to solve the VRP. Hoogeboom et al. (2020) propose an exact polynomial-time algorithm that can efficiently identify the optimal start time for serving each customer in the vehicle routing problem with multiple time windows.

Mathematical formulations for the cumulative vehicle routing problem with soft and hard time window constraints were presented by Fernández Gil et al. (2023). Their approach integrates $CO_2$ emissions into routing decisions, achieving a balance between environmental impact and time window compliance. Ulmer et al. (2024) demonstrate how companies can provide reliable, narrow time windows despite arrival time uncertainty by decoupling time window decisions from routing. Assuming specific arrival time distributions, they reduce a complex stochastic optimization problem to a one-dimensional root-finding task, enabling a practical heuristic for broader applications. Köhler et al. (2020) propose customer acceptance mechanisms for flexible time window management in attended home deliveries. Through a computational study using demand scenarios, including real data from a German online supermarket, they assess the approaches' effectiveness in offering short time windows while serving many customers.

Metaheuristic algorithms can be seen as versatile strategies applicable across diverse problem domains, offering iterative guidance and adjustment of subordinate heuristics. These methods intelligently blend various techniques to explore and exploit solution spaces, adapting heuristics to specific challenges. At each iteration, metaheuristics can manipulate either a single complete or incomplete solution, or a collection of solutions. Typical representatives of these algorithms are simulated annealing, tabu search, genetic algorithms, Variable Neighborhood Search (VNS) and Greedy Randomized Adaptive Search Procedure (GRASP). Simulated annealing is a robust optimization technique utilized across various problem domains. It draws inspiration from thermodynamics, specifically the gradual cooling process akin to metal cooling. This approach mimics the transformation of liquid metal into a crystal through slower cooling, symbolizing the exploration of the solution space from feasibility to discovering global optima. The seminal works by Kirkpatrick et al. (1983) and Černý (1985) introduced simulated annealing as a powerful algorithm for addressing challenging optimization problems. Notably, both studies applied simulated annealing to tackle the Traveling Salesman Problem in combinatorial optimization.

In 1995, Kontoravdis and Bard introduced a greedy randomized adaptive search procedure (GRASP) for solving the Vehicle Routing Problem with Time Windows (VRPTW). The approach minimizes the number of vehicles used and the total distance traveled. Lau et al. (2003) present a tabu search approach for the m-VRPTW, where a limited number of vehicles is given. The method uses a holding list, forces dense packing within routes, and allows time window relaxation through penalties. Computational results show competitive solutions for VRPTW and near-optimal results for m-VRPTW. Baños et al. (2013) proposed a multiobjective approach using simulated annealing and multiple-temperature Pareto simulated annealing to tackle the VRPTW, incorporating travel distance and route balance considerations. Adewumi and Adeleke (2018) review four most-addressed vehicle routing problem variations: capacitated, time windows, periodic, and dynamic, and examine their formulations, solution methods, and applications. A multi-objective approach combined with genetic algorithms is used by Ombuki and Hanshar (2006). The authors applied a multi-objective genetic algorithm to solve the VRPTW, optimizing both the number of vehicles and total cost without the need for predetermined weights. Their method produced competitive solutions compared to existing approaches and found new solutions that were not biased towards either the number of vehicles or cost. Schneider (2016) introduced a tailored tabu search algorithm for VRP with time windows, accommodating driver-specific familiarity with customers. Wang et al. (2015) focused on the Heterogeneous Multi-type Fleet Vehicle Routing Problem with time windows and incompatible loading constraints, developing heuristic and tabu search methods. In 2019, Marinakis et al. introduced a Multi-Adaptive Particle Swarm Optimization (MAPSO) algorithm to solve the VRPTW. The algorithm incorporates three adaptive strategies: using GRASP for initial solutions and iterations, an Adaptive Combinatorial Neighborhood Topology for particle movement, and dynamic parameter adaptation during execution. MAPSO was tested on benchmark instances and compared with other PSO versions and top algorithms, showing competitive performance in solving VRPTW instances with 100 to 1000 nodes.

Furthermore, Gehring and Homberger (2011) presented a cooperative parallel strategy using two-phase metaheuristics, combining evolutionary approaches to minimize vehicles followed by tabu search to optimize travel distance. Additionally, a cooperative parallel metaheuristic for VRPTW utilizes a solution warehouse strategy, facilitating the asynchronous exchange of best solution information among search threads (le Bouthillier and Crainic, 2005). Feng, Wei, and Hu (2023) proposed an Adaptive Large Neighborhood Search (ALNS) algorithm to solve the Vehicle Routing Problem with Multiple Time Windows (VRPMTW). This method uses an adaptive strategy to select neighborhoods and a local search based on destroy and repair operators to avoid local optima. Infeasible solutions are incorporated into the search, expanding the solution space, and an archive stores high-quality feasible solutions. In the study by Wang et al. (2024), an adaptive large neighborhood search (ALNS) algorithm for the multi-depot dynamic vehicle routing problem with time windows (MD-DVRPTW), where customer requests emerge stochastically, is developed. The ALNS includes novel removal operators and a time window compatibility-based insertion method, proving effective for dynamic problems requiring fast re-optimization. The study also highlights that vehicle fixed costs affect route planning, and that speeding up responsiveness can increase costs rather than improve the solution.

Agra et al. (2013) tackled the robust vehicle routing problem with time windows, focusing on maritime transportation where delays are common. It presents two new formulations based on different robust approaches: one using adjustable robust optimization and another based on path inequalities. Both approaches incorporate techniques to reduce the complexity of uncertainty. Comparative results show that the new formulations are faster than previous methods, achieving similar solution times while addressing the uncertainty in travel times effectively. Hu et al. (2018) examined a more realistic variation of VRPTW that involves demand and travel time uncertainty. To tackle large instances, they designed a two-stage method based on a modified variable neighborhood search heuristic. In Jabali et al. (2012) the authors introduce a new variant of VRP that accounts for travel time, fuel, and CO2 emissions. They incorporate CO2 emissions in routing decisions, relating CO2 to vehicle speed and fuel consumption, which changes during the day with respect to congestion. The conclusion is that it is important to control vehicle speed from a total cost point of view. Authors study the trade-off between minimizing CO2 emissions and minimizing total travel times. In his thesis, Souza (2024) introduces Flexi-VNS, a General Variable Neighborhood Search (GVNS) algorithm for solving three electric vehicle routing problems: EVRP, BSS-EV-LRP, and E-VRPTW. It employs a Randomized Variable Neighborhood Descent (RVND) method for local search and is shown to improve several best-known solutions. Flexi-VNS performs competitively, surpassing existing algorithms in multiple instances, including reducing battery swap stations and achieving high success rates.

Cattaruzza et al. (2017) review urban vehicle routing challenges in goods distribution, analyzing logistics flows and highlighting key issues: time-dependency, multi-level and multi-trip distribution, and dynamic information management. In Zhou et al. (2022), the authors investigate a novel variant of the two-echelon VRP incorporating time windows, pickup, and delivery, demonstrating its applicability in multimodal urban distribution, particularly for critical sectors such as medical supplies. The paper by Gutierrez et al. (2024) explores a two-echelon VRP incorporating time-dependent travel times to address urban traffic congestion. They employ a blend of soft and hard time windows to enhance customer service quality. The body of research on the Location-routing Problem is expanding. In their study, Mara et al. (2021) conducted a review of LRP literature, examining factors such as publication trends, problem characteristics, solution approaches, and application domains. A mixed-integer linear program was developed by Tian and Hu (2023) to minimize the distribution costs for a two-echelon location routing problem with recommended satellite facilities. A two-echelon LRP model was developed by Chen et al. (2024) to minimize total costs for a perishable food distribution network, accounting for the different needs of in-store pickup and delivery customers. In Bala et al. (2017), the authors investigate synchronization between production times and customer time windows to optimize service levels effectively. The approach was tested for a newspaper delivery problem. Yildiz et al. (2023), integrate location decisions into a two-echelon VRP that includes pickup and delivery operations. They apply this approach to a two-echelon supermarket chain, demonstrating its suitability for addressing real-world logistics challenges effectively. Escobar-Varbas and Crainic (2024) tackle a two-echelon location routing incorporating time-dependent multicommodity origin-to-destination demand, time windows, limited storage at intermediate facilities, and synchronization of fleets across echelons. This new problem, involves selecting facilities, routing vehicles, scheduling synchronized operations at intermediate sites, and allocating demands using time-sensitive routes.

## 2. Problem formulation

In this section we will introduce VRPTW in more details. Let $\mathcal{C} = \{1, \dots, n\}$ be a set of customers and $r_i$ a request of a customer $i$ in $\mathcal{C}$ , that has to be served within time window $[t_i^a, t_i^d]$ with a serving time $t_i^s$. The time dimension besides the customers is also associated with depot, so that $D$ has also defined window $[t_D^d, t_D^a]$. Service is carried with a homogenous fleet $\mathcal{V}$ of capacity $Q$. Each vehicle leaving the depot, starts distribution in time $t_D^d$ and returns before the closing time $t_D^a$. At customers, vehicle cannot start service before the start of time window $t_i^a$ and if arrives sonner than $t_i^a$ it has to wait. The arrival time at customer $i$ cannot be after the end time $t_i^d$. The serving time $t_i^s$ defines holding time at customer $i$.

Let $G = (N, A)$ be an oriented graph defined on a set of vertices $N = \{0, 1, \dots n, n + 1\}$ so that vertices 0 and $n + 1$ represent depot, and $1, \dots n$ is the customer set $\mathcal{C}$. Arcs $A$ of the graph are generated by connections between customers $\mathcal{C}$, while arcs with depot are modelled so that vertex

$0 \rightarrow \{1, \dots, n\}$ and represents leaving arcs from depot, while $\{1, \dots, n\} \rightarrow n+1$ and represents arcs entering the depot. For $i \in N$, $O_i$ represents the outset $O_i = \{j : (i,j) \in A\}$, and $I_i$ is the inset $I_i = \{j : (j,i) \in A\}$.

In the presented model we have two types of variables: binary $x_{ij}^V$ and continuous $\omega_i^V$. Binary variable is defined as 1 if a vehicle $V$ uses the arc $(i,j)$, and 0 otherwise. Continuous variable $\omega_i^V$ represents time, more precisely start of serving time at customer $i$ by a vehicle $V$. Time window associated with depot, that corresponds to vertices 0 and $n+1$, is $\left[t_0^d, t_0^a\right] = \left[t_{n+1}^d, t_{n+1}^a\right] = [t_D^d, t_D^a]$.

$$min \left\{ \sum_{V \in \mathcal{V}} \sum_{j \in O(0)} x_{0j}^V, \sum_{V \in \mathcal{V}} \sum_{(i,j) \in A} c_{ij} x_{ij}^V \right\} \qquad (1)$$

$$\sum_{V \in \mathcal{V}} \sum_{j \in N} x_{ij}^V = 1 \qquad \forall i \in N \backslash \{0, n+1\} \qquad (2)$$

$$\sum_{j \in O(0)} x_{0j}^V \le 1 \qquad \forall V \in \mathcal{V} \qquad (3)$$

$$\sum_{j \in O(i)} x_{ij}^V - \sum_{j \in I(i)} x_{ji}^V = 0 \qquad \forall V \in \mathcal{V}, i \in N \backslash \{0, n+1\} \qquad (4)$$

$$\sum_{j \in O(0)} x_{0j}^V - \sum_{j \in I(n+1)} x_{j,n+1}^V = 0 \qquad \forall V \in \mathcal{V} \qquad (5)$$

$$x_{ij}^V (\omega_i^V + t_i^s + t_{ij} - \omega_j^V) \le 0 \qquad \forall V \in \mathcal{V}, (i,j) \in A \qquad (6)$$

$$t_i^a \sum_{j \in O(i)} x_{ij}^V \le \omega_i^V - t_i^d \sum_{j \in O(i)} x_{ij}^V \qquad \forall V \in \mathcal{V}, i \in N \backslash \{0, n+1\} \qquad (7)$$

$$t_i^d \le \omega_i^V \le t_i^a \qquad \forall V \in \mathcal{V}, i \in \{0, n+1\} \qquad (8)$$

$$\sum_{i \in N\{0,n+1\}} r_i \sum_{j \in O(i)} x_{ij}^V \le Q \qquad \forall V \in \mathcal{V} \qquad (9)$$

$$x_{ij}^V \in \{0,1\} \qquad \forall V \in \mathcal{V}, (i,j) \in A \qquad (10)$$

$$\omega_i^V \ge 0 \qquad \forall V \in \mathcal{V}, i \in N \qquad (11)$$

The objective function (1) contemplates two objectives: number of vehicles and travel costs. With (2) we assure that each customer is visited exactly once. Inequality (3) implies that each vehicle is used exactly once, and combined with (4) and (5) defines the route of vehicle $V \in \mathcal{V}$. With (6)-(8) and (9) we follow the time constraints and capacity constraint, respectively. For a vehicle $V \in \mathcal{V}$ (7) implies $\omega_i^V = 0$ if customer $i$ is not served by a vehicle $V$. Finally, (10) and (11) are variable constraints.

## 3. The algorithm

In this section, we present an algorithm for VRPTW that runs in predefined time blocks. Within each block, our primary objective is to evaluate the total travel costs. However, some blocks are treated as "special", where the algorithm randomly selects a route $s$ to focus on. Let $m$ be the number of customers on route $s$, and $w$ the cumulative quantity of all customers on $s$. Additional evaluation calculates the logarithm of $m$ and average of $w$. The evaluation function is structured to prioritize moving customers away from the route by the means of a logarithmic function. The average of quantities tends to keep customers with smaller requests.

With appropriate term hierarchy, the algorithm will prioritize removing customers from the route first; if that's not possible, it will retain customers of smaller quantities who have a greater number of potential positions. If the number of customers reaches zero, and the running time for the block hasn't elapsed, the new route is chosen randomly. As the running time of the algorithm increases, we decrease the probability of declaring the special blocks, and gradually focus only on travel costs.

Let $F_1$ be the number of vehicles and $F_2$ the travel costs. Then the integral form of the evaluation function can be written as: $E(\cdot) = \alpha \cdot F_1 + \beta \cdot F_2 + \gamma \cdot \log(m+1) + \delta \frac{w}{m+1}$. If a block is not special, $\gamma = \delta = 0$. Otherwise , $\delta > 0$ .

Exploration of the solution space is executed using the simulated annealing technique. As it is shown in several papers, for instance see Chiang and Russell (1996), Wang et al. (2015), Baños et al. (2013), Bala et al. (2024), this metaheuristic can return solutions of high quality. In the review by Konstantakopoulos et al. (2022) the method is emphasized as one of the approaches that can effectively overcome the algorithm being stuck in local optimum. In this research we decided to apply the simulated annealing because of its robustness and relatively easy manipulation of the evaluation function. Another feature that we apply is guiding the search through infeasibility regions. During the special blocks, we accept solutions with minor time window or vehicle capacity violations. The idea is simple: to overcome local minima.

Generation of potential neighboring solutions is carried out via four transformations. To give a brief explanation, we will use the oriented graph described in a previous section. More comprehensive overview can be found in Bräysy and Gendreau (2005a).

1. Move: choose vertex $u \in M$ and $v \in \{0\} \cup M$, and move $u$ after $v$. See Figure 1.
2. Swap: choose vertices $u, v \in M$, and swap their positions. See Figure 2.
3. 2-opt: choose vertices $u, v \in M$ on the same route and change the direction in the sub-route that they define. See figure 3.
4. Cross: choose vertices $u, v \in M$ on different routes and swap their remaining tails. See figure 4.

The Figures 1 – 4 are graphical presentations of transformation 1 – 4, respectively. The square indicates Depot, the start and the end of the routes, while circles represent customers. Direction of routes are indicated with arrows.

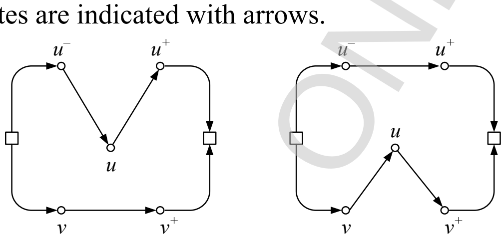

**Figure 1** Move transformation
**Source:** The authors

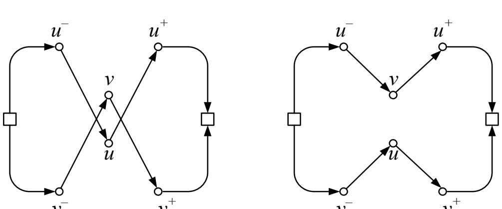

**Figure 2** Swap transformation
**Source:** The authors

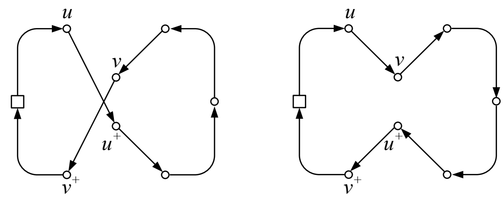

**Figure 3** 2-opt transformation
**Source:** The authors

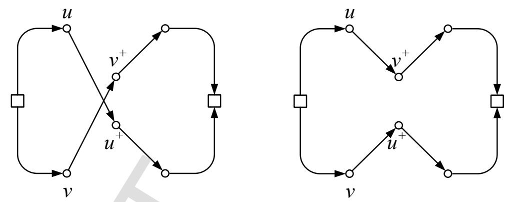

**Figure 4** Cross transformation
**Source:** The authors



## 4. Results

This section contains the results of the proposed algorithm. We evaluated its performance using the Gehring and Homberger benchmark instances designed for 200 customers (https://www.sintef.no/projectweb/top/vrptw/200-customers/). These test instances are extensions of the well-known Solomon test problems for VRPTW and follow the same topology differentiation: clustered (C), random (R), and mixture (RC), and time scheduling horizon: short (1) and long (2). Each class (topology, horizon) contains 10 instances, resulting in 60 test instances.

For simulated annealing we define start and end temperatures of 20, and 0,5 respectively, with a cooling factor 0,999999. Each block lasts for 5 seconds, and applying the cooling rule $T \coloneqq T \cdot 0{,}999999$ leads to the running time of the algorithm to being approximately 15 minutes (915 seconds).

For each problem instance, the algorithm is executed 10 times. In Table 1 we present a summary of the test results categorized by instance group. The first column denotes the problem group, followed by the deviation of the best-solution $(bs)$ found from the best known solution $(bks)$. The deviation is calculated as $(bs - bks)/bks$ and reported as percentage. Then we take the best among the whole group. The third column contains the average of deviations for each obtained solution after 10 runs. We report the average over the respective group.

Motivated by practitioner evaluation of results, where the tradeoff between execution time and solution quality, or commonly small improvements that the algorithm makes in the latest part of execution, we introduce the measure "class" for each solution. If the deviation from the best-known solution is less than $(p+1)\%$, we call the solution to be of class $p$. Average class for test results are given in the third column.

**Table 1** Summary of the results across problem groups

| Problem group | Minimum (%) | Average (%) | Average class |
|---|---|---|---|
| c1 | 0,0000 | 0,2256 | 0,1 |
| c2 | 0,0005 | 0,1747 | 0,1 |
| r1 | 0,0000 | 0,6244 | 0,2 |
| r2 | 0,0005 | 0,5697 | 0,1 |
| rc1 | 0,4579 | 2,3313 | 1,9 |
| rc2 | 0,0351 | 1,1484 | 0,7 |
| Total | 0,0000 | 0,8457 | 0,5 |

**Source:** The authors

From the final row, we can see that our algorithm creates solutions that are on average 0,8457% behind the best solutions reported in the literature. This average is calculated across 600 test cases. The algorithm matches the best solutions in 23 tests, and in Table 2 we give a distribution of test cases among the classes. Specifically, 414 or 69% of test cases are within 1% of the best-known solution. Moreover, in 572 or 95,33% of cases our results lag by no more than 3%, demonstrating the algorithm's consistency and robustness.

**Table 2** Class distribution

| Class | Relative frequency (%) |
|---|---|
| 1 | 69,00 |
| 2 | 18,83 |
| 3 | 7,50 |
| 4 | 2,83 |
| 5 | 1,17 |
| 6 | 0,33 |
| 7 | 0,33 |
| Total | 100,00 |

**Source:** The authors

Time consumption is presented in Table 3, with averages calculated within each respective group. The second column details the average time required to reach the minimum number of vehicles. In the third column, we report the average time taken to achieve the best solution for each test instance. Finally, the fourth column shows the average time required to attain the class for each test.

**Table 3** Time consumption

| Problem group | Average time (vehicles) | Average time (best) | Average time (class) |
|---|---|---|---|
| c1 | 26,3 | 650,8 | 449,7 |
| c2 | 18,0 | 750,9 | 506,2 |
| r1 | 21,1 | 692,3 | 547,4 |
| r2 | 24,5 | 737,1 | 560,4 |
| rc1 | 34,6 | 798,4 | 624,9 |
| rc2 | 34,4 | 822,9 | 641,7 |
| Total | 26,5 | 742,1 | 555,0 |

**Source:** The authors

Note that although the algorithm reaches the minimum number of vehicles relatively quickly, it continues to periodically run in special blocks. That is not optimal behavior from a running time perspective. Furthermore, from Table 3, we can see that the algorithm spends on average an additional 187 seconds in making small progress in transportation costs from its class to the best solution. There is potential to make further improvements in running time efficiency in this area.



## Conclusion

In this paper, we address the Vehicle Routing Problem with Time Windows (VRPTW), a critical component of logistics models. We present an algorithm for VRPTW designed to operate in predefined time blocks. Within each block, our primary objective is to minimize travel costs. Additionally, in certain blocks, we focus on reducing the number of vehicles involved by randomly selecting routes and including the number of requests and their quantities as part of our evaluation strategy. In overcoming the local optimum, we let algorithm explore the infeasible regions, from time and capacity perspective.

Our approach has demonstrated high quality solutions on Gehring and Homberger benchmark instances featuring 200 customers. Across all instances, we achieve the minimum number of vehicles reported in the literature. With travel costs averaging 0,8457% from the best-known solutions, we affirm the effectiveness of our algorithm.

Given the integral role of VRPTW in logistics software, we analyze solutions also from a practical standpoint. Our approach emphasizes achieving reasonably good solutions within feasible time frames, categorizing solutions into performance classes. Specifically, 69% of test cases are within 1% of the best-known solution, and 95% are within 3%.

While our method excels in solution quality, time efficiency remains a challenge. We find that 20% of execution time is spent on marginal improvements, indicating space for enhancing runtime efficiency. Future research will focus on refining our algorithm through advanced transformations, parallelization, and additional experimentation in evaluation functions to achieve faster execution times and further improve solution quality.

## Declarations

## Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

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## References

Agra, A., Christiansen, M., Figueiredo, R., Hvattum, L. M., Poss, M., & Requejo, C. (2013). *The robust vehicle routing problem with time windows. Computers & operations research, 40*(3), 856-866. https://doi.org/10.1016/j.cor.2012.10.002

Adewumi, A. O., & Adeleke, O. J. (2018). A survey of recent advances in vehicle routing problems. *International Journal of System Assurance Engineering and Management, 9*(1), 155–172. https://doi.org/10.1007/s13198-016-0493-4

Bala, K., Brcanov, D., & Gvozdenović, N. (2017). Two-echelon location routing synchronized with production schedules and time windows. *Central European Journal of Operations Research*, *25*(3), 525-543. https://doi.org/10.1007/s10100-016-0463-6

Bala, K., Bandecchi, M., Brcanov, D., & Gvozdenović, N. (2024). Optimizing vehicle routing and scheduling under time constraints. *Anali Ekonomskog Fakulteta u Subotici, 52*, 109–118. https://doi.org/10.5937/AnEkSub2400005B

Baños, R., Ortega, J., Gil, C., Fernández, A., & de Toro, F. (2013). A Simulated Annealing-based parallel multi-objective approach to vehicle routing problems with time windows. *Expert Systems with Applications*, *40*(5), 1696-1707. https://doi.org/10.1016/j.eswa.2012.09.012

Bräysy, O., & Gendreau, M. (2005a). Vehicle routing problem with time windows, Part I: Route construction and local search algorithms. *Transportation Science*, *39*(1), 104-118. https://doi.org/10.1287/trsc.1030.0056

Bräysy, O., & Gendreau, M. (2005b). Vehicle routing problem with time windows, Part II: Metaheuristics. *Transportation Science*, *39*(1), 119-139. https://doi.org/10.1287/trsc.1030.0057

le Bouthillier, A., & Crainic, T. G. (2005). A cooperative parallel meta-heuristic for the vehicle routing problem with time windows. *Computers and Operations Research, 32*(7), 1685–1708. https://doi.org/10.1016/j.cor.2003.11.023

Cattaruzza, D., Absi, N., Feillet, D., & González-Feliu, J. (2017). Vehicle routing problems for city logistics. *EURO Journal on Transportation and Logistics, 6*(1), 51–79. https://doi.org/10.1007/s13676-014-0074-0

Chen, X., Jiu, Y., & Hu, D. (2024). Two-Echelon Location–Routing Problem of Perishable Products Based on the Integrated Mode of In-Store Pick-Up And Delivery. *Transportation Research Record: Journal of the Transportation Research Board, 2678*(8), 622–636. https://doi.org/10.1177/03611981231218008

Chiang, W. C., & Russell, R. A. (1996). Simulated annealing metaheuristics for the vehicle routing problem with time windows. *Annals of Operations Research*, *63*, 3-27. https://doi.org/10.1007/BF02601637

Christofides, N., & Beasley, J. E. (1984). The period routing problem. *Networks*, *14*(2), 237-256. https://doi.org/10.1002/net.3230140205

Clarke, G., & Wright, J. W. (1964). Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. *Operations Research*, *12*(4), 519-643. https://doi.org/10.1287/opre.12.4.568

Černý, V. (1985). Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm. *Journal of Optimization Theory and Applications*, *45*(1), 41-51. https://doi.org/10.1007/BF00940812

Dantzig, G. B., & Ramser, J. H. (1959). The Truck Dispatching Problem. *Management Science*, *6*(1), 80-91. https://doi.org/10.1287/mnsc.6.1.80

Desrochers, M., Desrosiers, J., & Solomon, M. (1992). A new optimization algorithm for the vehicle routing problem with time windows. *Operations Research*, *40*(2), 342-354. https://doi.org/10.1287/opre.40.2.342

Desrosiers, J., Soumis, F., & Desrochers, M. (1984). Routing with time windows by column generation. *Networks*, *14*(4), 545-565. https://doi.org/10.1002/net.3230140406

Gutierrez, A., Labadie, N., Prins, C. (2024). A Two-echelon Vehicle Routing Problem with time-dependent travel times in the city logistics context. *EURO Journal on Transportation and Logistics*, *13*(2024) 1-22. https://doi.org/10.1016/j.ejtl.2024.100133

Escobar-Vargas, D., & Crainic, T. G. (2024). Multi-attribute two-echelon location routing: Formulation and dynamic discretization discovery approach. *European Journal of Operational Research*, *314*(1), 66-78. https://doi.org/10.1016/j.ejor.2023.09.031

Feng, B., Wei, L., & Hu, Z. (2023). An adaptive large neighborhood search algorithm for Vehicle Routing Problem with Multiple Time Windows constraints. *Journal of Industrial and Management Optimization (JIMO), 19*(1), 573-593. https://doi.org/10.3934/jimo.2021197

Fernández Gil, A., Lalla-Ruiz, E., Gómez Sánchez, M., & Castro, C. (2023). The cumulative vehicle routing problem with time windows: models and algorithm. *Annals of Operations Research*. https://doi.org/10.1007/s10479-022-05102-7

Fisher, M. L., Jörnsten, K. O., & Madsen, O. B. G. (1997). Vehicle routing with time windows: Two optimization algorithms. *Operations Research, 45*(3), 488-492. https://doi.org/10.1287/opre.45.3.488

Homberger, J., & Gehring, H. (2005). A two-phase hybrid metaheuristic for the vehicle routing problem with time windows. *European Journal of Operational Research*, *162*(1), 220-238. https://doi.org/10.1016/j.ejor.2004.01.027

Hoogeboom, M., Dullaert, W., Lai, D., & Vigo, D. (2020). Efficient Neighborhood Evaluations for the Vehicle Routing Problem with Multiple Time Windows. *Transportation Science, 54*(2), 400–416. https://doi.org/10.1287/trsc.2019.0912

Hu, C., Lu, J., Liu, X., & Zhang, G. (2018). Robust vehicle routing problem with hard time windows under demand and travel time uncertainty. *Computers and Operations Research, 94*, 139-153. https://doi.org/10.1016/j.cor.2018.02.006

Jabali, O., van Woensel, T., & de Kok, A. G. (2012). Analysis of travel times and CO2 emissions in time-dependent vehicle routing. *Production and Operations Management*, *21*(6), 1060-1074. https://doi.org/10.1111/j.1937-5956.2012.01338.x

Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, *220*(4598), 671-680. https://doi.org/10.1126/science.220.4598.671

Kohl, N., & Madsen, O. B. G. (1997). An optimization algorithm for the vehicle routing problem with time windows based on Lagrangian relaxation. *Operations Research*, *45*(3), 395-406. https://doi.org/10.1287/opre.45.3.395

Köhler, C., Ehmke, J. F., & Campbell, A. M. (2020). Flexible time window management for attended home deliveries. *Omega, 91*(2020), 102023. https://doi.org/10.1016/j.omega.2019.01.001

Kontoravdis, G., & Bard, J. F. (1995). A GRASP for the vehicle routing problem with time windows. ORSA journal on Computing, 7(1), 10-23. https://doi.org/10.1287/ijoc.7.1.10

Kolen, A. W. J., A. H. G. Rinnooy Kan, & H. W. J. M. Trienekens. (1987). Vehicle Routing with Time Windows. *Operations Research*, 35(2), 266–273. http://www.jstor.org/stable/170698

Konstantakopoulos, G. D., Gayialis, S. P., & Kechagias, E. P. (2022). Vehicle routing problem and related algorithms for logistics distribution: a literature review and classification. *Operational Research*, *22*(3), 2033–2062. https://doi.org/10.1007/s12351-020-00600-7

Laporte, G. (1992). The vehicle routing problem: An overview of exact and approximate algorithms. *European journal of operational research, 59*(3), 345-358. https://doi.org/10.1016/0377-2217(92)90192-C

Lau, H. C., Sim, M., & Teo, K. M. (2003). Vehicle routing problem with time windows and a limited number of vehicles. *European journal of operational research, 148*(3), 559-569. https://doi.org/10.1016/S0377-2217(02)00363-6

Li, N., & Wang, Z. (2025). Vehicle routing problem for omnichannel retailing including multiple types of time windows and products. *Computers & Operations Research, 173*, 106828. https://doi.org/10.1016/j.cor.2024.106828

Lenstra, J. K., & Kan, A. H. G. R. (1981). Complexity of vehicle routing and scheduling problems. *Networks*, *11*(2), 221-227. https://doi.org/10.1002/net.3230110211

Mara, S. T. W., Kuo, R. J., & Asih, A. M. S. (2021). Location-routing problem: a classification of recent research. International Transactions in Operational Research, 28(6), 2941–2983. https://doi.org/10.1111/itor.12950

Marinakis, Y., Marinaki, M., & Migdalas, A. (2019). A multi-adaptive particle swarm optimization for the vehicle routing problem with time windows. *Information Sciences, 481*, 311-329. https://doi.org/10.1016/j.ins.2018.12.086

Ombuki, B., Ross, B. J., & Hanshar, F. (2006). Multi-objective genetic algorithms for vehicle routing problem with time windows. *Applied Intelligence, 24*, 17-30. https://doi.org/10.1007/s10489-006-6926-z

Schneider, M. (2016). The vehicle-routing problem with time windows and driver-specific times. *European Journal of Operational Research*, *250*(1), 101-119. https://doi.org/10.1016/j.ejor.2015.09.015

Solomon, M. M. (1987). ALGORITHMS FOR THE VEHICLE ROUTING AND SCHEDULING PROBLEMS WITH TIME WINDOW CONSTRAINTS. *Operations Research, 35*(2), 254-265. https://doi.org/10.1287/opre.35.2.254

Souza, A. L. D. S. (2024). A variable neighborhood search algorithm for a class of electric vehicle routing problems. https://www.repositorio.ufop.br/handle/123456789/18939

Tian, X.-D., & Hu, Z.-H. (2023). A branch-and-price method for a two-echelon location routing problem with recommended satellites. *Computers & Industrial Engineering, 184*, 109593. https://doi.org/10.1016/j.cie.2023.109593

Ulmer, M. W., Goodson, J. C., & Thomas, B. W. (2024). Optimal Service Time Windows. Transportation Science, 58(2), 394–411. https://doi.org/10.1287/trsc.2023.0004

Wang, C., Mu, D., Zhao, F., & Sutherland, J. W. (2015). A parallel simulated annealing method for the vehicle routing problem with simultaneous pickup-delivery and time windows. *Computers and Industrial Engineering*, *83*(May 2015), 111-122. https://doi.org/10.1016/j.cie.2015.02.005

Wang, S., Sun, W., & Huang, M. (2024). An adaptive large neighborhood search for the multi-depot dynamic vehicle routing problem with time windows. *Computers & Industrial Engineering, 191*, 110122. https://doi.org/10.1016/j.cie.2024.110122

Yıldız, E. A., Karaoğlan, İ., & Altiparmak, F. (2023). An exact algorithm for Two-Echelon Location-Routing problem with simultaneous pickup and delivery. *Expert Systems with Applications*, *231*, 120598. https://doi.org/10.1016/j.eswa.2023.120598

Zhou, S., Zhang, D., Ji, B., & Li, S. (2024). Two-echelon vehicle routing problem with direct deliveries and access time windows. *Expert Systems with Applications*, *244*, 121150. https://doi.org/10.1016/j.eswa.2023.121150

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