STRATEGIC MANAGEMENT, Vol. XX (202x), No. x, pp. 0xx-0xx

DOI: 10.5937/StraMan2400019S

Received: July 08, 2024
Revised: October 23, 2024
December 18, 2024
Accepted: January 14, 2025
Published online: February 23, 2025

The Application of Stochastic ICIM Model in the Decision-Making Processes of Insurance Product Management

Zsolt Simonka

University of Economics in Bratislava, Faculty of Economic Informatics, Department of Mathematics and Actuarial Science, Bratislava, Slovak Republic
https://orcid.org/0000-0002-4451-667X

Ingrid Krčová

University of Economics in Bratislava, Faculty of Economic Informatics, Department of Mathematics and Actuarial Science, Bratislava, Slovak Republic
https://orcid.org/0009-0000-4810-5299

Veronika Horniaková

University of Economics in Bratislava, Faculty of Economic Informatics, Department of Applied Informatics, Bratislava, Slovak Republic
https://orcid.org/0009-0000-9621-3905

Abstract

Background: The significance of this study arises from the increasing complexity of managing insurance products, driven by the need to accurately model and predict the occurrence of insured events and associated risks. These processes are relevant not only to life insurance companies but to any institution offering personal insurance and supplementary coverages, such as banks, brokerage firms, and others. Existing literature highlights extensive possibilities for the application of stochastic processes in various fields, including finance, biology, and environmental engineering, with notable applicability in insurance.
Purpose: This article aims to explore the application of stochastic models in the decision-making processes for managing insurance products. Specifically, it focuses on the development and utilization of multi-state models for pricing selected insurance products and analysing the impact of parameter changes on the amount of regular net premium.
Study design/methodology/approach: We start with the traditional 'Healthy-Dead' model, which we extend to include the 'Sick' state. By restricting the transition from this state to the 'Healthy' state, we obtain a three-state 'Healthy-Sick-Dead' model for incurable critical illness. This is a non-homogeneous Markov process characterized by the respective transition probabilities. Actuarial calculations of transition probabilities are based on specific statistical data from an unnamed insurance company. The resulting regular net premium represents the real (net) price of the supplementary insurance product for incurable critical illness.
Findings/conclusions: The main findings suggest that incorporating stochastic models into the creation and management of insurance products allows for more accurate predictions of insured events and better risk assessment. The introduced three-state model provides a robust framework for pricing supplementary insurance due to incurable critical illness. The analysis demonstrates how changes in transition probabilities affect the amount of net premium, underscoring the importance of precise parameter estimation.
Limitations/future research: The study's limitations include reliance on accurate historical data, which may not fully capture future trends and changes in health outcomes, as experienced during the Covid-19 pandemic. Future research should explore the integration of a larger amount of real data and advanced computational methods for their processing. Additionally, extending the model to include the 'recovery' transition would enhance its applicability for 'all' types of critical illnesses. The creation of such an insurance product would, however, assume the availability of a large amount of high-quality data (Schmidt, 2021).

Keywords
Stochastic processes, insurance, rider, multi-state models, critical illness, transition probabilities, Markov processes, annual net premium, Incurable Critical Illness Model

Introduction

The turn of the millennium marked a significant expansion in the use of stochastic processes, with extensive applications in modelling financial markets, predicting stock prices and commodity prices, developing financial models and risk management strategies (Shreve, 2004; Cerqueti, 2021), in operations research (Campbell & MacKinlay, 1997; Fu & Hu, 2002), in modelling biological systems (Wilkinson, 2006; Rogers, 2011), and in the development of prognostic models in medicine (Simpson, et al., 2012; Bergés, et al., 2023), biometrics (Hu & Laurière, 2023) and biophysics (He, 2022). Stochastic processes also play a key role in hydrology and environmental engineering for modelling weather conditions, weather forecasting (Weeks, 2010), and climate change analysis and modelling (Palmer, 1999; Hipel, 2000). In geophysics, they are used in modelling geological processes, analysing earth layers (Gautier, 2016), and modelling groundwater contamination (Neuman & Wierenga, 2012).

Currently, stochastic processes remain a focus of active research. New technologies and computational methods allow for more sophisticated analyses and modelling of random processes across many domains. These analyses often employ advanced mathematical and computational techniques to solve complex problems and gain a deeper understanding of the structure and behaviour of random processes. Their flexibility and ability to model unpredictable phenomena make them an indispensable tool in various scientific disciplines, including insurance. In this field, stochastic models are crucial for pricing insurance products and managing risks associated with events such as accidents, natural disasters, or critical illnesses.

Given the constant advancements in medicine and changes in population health outcomes, it is essential to continually adapt and improve existing models to better reflect current conditions and risks. In this context, exploring new approaches to modelling insurance risks is imperative, and this was the motivation behind conducting this study.

The aim of this study is to develop and apply a three-state stochastic process model, called the Incurable Critical Illness Model (ICIM), for assessing premiums for incurable critical illnesses in life insurance. This model extends the traditional two-state 'Healthy–Dead' model by including a 'Sick' state, allowing for a more accurate reflection of health risks and enabling more precise premium determination for products that include critical illness riders.

The proposed method is based on stochastic process theory, as described by Shreve (2004) and Bowers et al. (1997) in the context of actuarial mathematics. A Markov process is used to model transitions between health states, a technique that has already been applied in biostatistical and actuarial studies (Škrovánková & Simonka, 2021).

In this study, the ICIM model is applied to calculate net premiums for critical illness riders.

Based on the nature of this article, several practical research questions can be formulated:

1. How do the transition probabilities between health states affect the calculation of net premiums?
2. How might changes in the transition probabilities within the ICIM model impact the insurer's financial stability?

These research questions lead to an exploration of the relationship between health state transitions and their financial implications for the insurer, offering valuable insights for more accurate risk assessment and premium determination.

1. The Possibilities of Using Stochastic Processes and Modelling in Insurance

Stochastic processes are a key tool in the insurance industry, offering a wide range of applications. Insurance events, such as death due to accidents or other causes, changes in health status (e.g., contracting an incurable disease), accidents, fires, natural disasters, or other catastrophic events, are often random and irregular. To address these uncertainties, stochastic models are employed to estimate the probability of such events occurring over time. These processes are crucial for risk modelling, financial product valuation, and portfolio management. They also offer tools for forecasting future events and developing risk management strategies (Poláček & Páleš, 2012; Brokešová, et al, 2023). Their applications include the design of life insurance products and pension plans, minimising mortality and longevity risks (Choulli et al., 2021; D´Amato, et al., 2020), reinsurance (Colaneri, et al. 2024; Shen, 2024), and capital management.

1.1. Modelling Insurance Claims

Insurance claim modelling uses stochastic processes to predict insurance events. In life insurance, it helps forecast longevity and mortality, while in non-life insurance, it models the number of claims and total damages (Clemente et al., 2023; Tadayon & Ghanbarzadeh, 2024). This process is important for actuarial risk assessment and premium determination. Various models, such as the Poisson process and Binomial model, offer different advantages, making model selection crucial. Accurate claim prediction involves identifying key factors, configuring the model, estimating parameters using historical data, and evaluating performance. Insurers' ability to handle big data significantly impacts this process (Mišút, 2021).

1.2. Development of Insurance Products

Insurance companies offer products with varying returns and risks, and developing new products is a dynamic process aimed at meeting customer needs while ensuring the company's sustainability. Products are designed with goals such as increasing market share, competitiveness, profitability, or offering new protection options. This process involves defining coverage, terms, premiums, and policy duration, followed by actuarial and financial analyses to assess risk and set appropriate premiums. After launch, product performance is monitored and adjusted as necessary. Successful product development requires market analysis, understanding customer needs, and adapting to industry trends (Hellriegel, 2019).

1.3. Risk Management

Risk management involves identifying, assessing, and managing risks that impact an organization's operations and performance. Stochastic processes help insurance companies quantify risks within their insurance portfolios, ensuring solvency and stability. Risks can be internal (operational, HR, technological) or external (market, competition, natural). Risk assessment determines the likelihood and impact of each risk, helping organizations prioritize those that require management. Effective risk management is essential for long-term success, financial stability, and solvency in uncertain environments, leading to improved performance and value protection (McNeil, Frey & Embrechts, 2015).

1.4. Customizable Insurance Premium Rates

Customizable insurance premium rates allow companies to better account for individual risks and customer needs, personalizing prices based on factors like age, gender, residence, driving habits, and health status. This approach is gaining popularity due to increased data availability and advancements in data analysis (Páleš, 2012). Data collection and dynamic pricing are essential, enabling companies to adjust rates based on real-time data. For example, a driver who frequently uses safety features may pay less than one with a poor driving record (Biener, 2019). Stochastic models help ensure fair, risk-based premium rates.

2. Multi-state Models in Applications

Multi-state models (MsM) represent a powerful tool for analysing and predicting the behaviour of complex systems. They are utilised in various scientific and applied fields (some of which are mentioned below) to model the interactions between different parts of a system and to understand the dynamics of these systems.

2.1. Economics

In economics, multi-state models are used to model economic systems, including market interactions, macroeconomic trends, and the behaviour of consumers and producers. These models can help in predicting market developments, analysing economic policy, and assessing economic risks.

One specific example of the use of multi-state models in economics is the modelling of economic cycles using Dynamic Stochastic General Equilibrium (DSGE) models.

In addition to DSGE models, multi-state models are also used to analyse the impact of external shocks, such as pandemics, on the economy. For example, the epidemic–economic delay model has been applied to study the effects of lockdowns on the progression of infectious diseases and economic performance. The model emphasises that well-timed and strict lockdowns can significantly reduce infection rates and delay the epidemic peak, thereby easing pressure on healthcare systems. However, such measures can also negatively impact the economy, particularly if isolated individuals become less productive (Ishikawa, 2022; Mozokhina et al., 2024).

2.2. Social Sphere

Multi-state models in social sciences are used to model social networks, the spread of opinions, and the behaviour of individuals and groups. These models can be useful in analysing the spread of diseases, the dissemination of information, and predicting trends in society.

An example of the use of a multi-state model in the social sphere is the modelling of opinion or behaviour spread in social networks. These models are used to study social phenomena such as trend propagation, political polarisation, group identity formation, or the spread of new technologies. They can also be used to predict social changes or to design strategies to influence social behaviour.

A specific practical example is the "Diffusion of Innovations Model", which is used to study the spread of new products, technologies, or innovations in a population (Rogers, 2003).

2.3. Insurance

Multi-state models are increasingly utilised in insurance to model the complex processes of changes in the health status of insured individuals and their life events. These models enable the analysis of transitions between various health and risk states, which is crucial for product pricing, risk management, and the establishment of insurance reserves (D'Amico, et al., 2017). In insurance, multi-state models are mathematical and statistical tools used to predict risk, de-risking (Levantesi, et al., 2024; D´Amato, et al., 2020) and determine the pricing of insurance products (Christiansen & Niemeyer, 2015). These models take into account not only traditional factors such as age, gender, and health status but also factors like geographic location, lifestyle, and their interactions (D'Amico, et al., 2020). They enable insurance companies to forecast the likelihood of various insurance events, utilising a wide range of statistical methods, probabilistic models, and historical data to predict the number of accidents, extent of damages from accidents, fires, natural disasters, and other events, as well as the spread of diseases and their fatal outcomes.

In the context of critical illness insurance, these models become particularly valuable. For instance, stochastic interest rate models, like the Cox-Ingersoll-Ross model, are used in combination with multi-state Markov models to simulate transitions between health states, such as healthy, critically ill, or deceased, allowing for more accurate premium calculations and risk assessment (Alim, et al., 2019). Similarly, the increasing use of multi-state models in public health, particularly during the Covid-19 pandemic, demonstrates their versatility (Mohammadi et al., 2024; Zhao et al., 2024). The SIRD model (susceptible, infected, recovered, dead), for example, helps estimate the basic reproduction number R_0, providing insights into virus transmission dynamics (Zuhairoh, et al., 2021). Additionally, the SVIRD model (susceptible, vaccinated, infected, recovered, dead), which incorporates vaccination status, has been instrumental in predicting multiple pandemic waves and improving the accuracy of epidemic forecasting (Zuhairoh, et al., 2024). These examples highlight the broad applicability of multi-state models in both insurance and public health, where they serve as critical tools for managing risk and uncertainty.

In the following sections, we will specifically focus on certain types of illnesses – incurable critical illnesses and their modelling.

3. Developing and Application of Multi-state Models of Stochastic Processes in Insurance

3.1. Methodology

The 'Healthy - Dead' two-state model is a simple mathematical model used in epidemiology or biostatistics to describe a population in which each individual is in one of two possible states: 'Healthy' or 'Dead' (Andersen & Ravn, 2023). The equivalent model 'Alive - Dead' (also known as the mortality model or the basic survival model) is the simplest multi-state model applied in insurance.

By extending this model to include the 'Critically Ill' state, we develop a three-state model aimed at modelling the product "Incurable Critical Illness Rider". We will evaluate the quality and completeness of the collected data to ensure their suitability for applying a multi-state stochastic model. Using theoretical insights from the modelling of stochastic processes with the Markov property, we will construct a three-state model and use it to calculate the premium for a client in a specific age category.

The three-state ICIM model offers significant advantages in the accuracy of premium calculation and risk modelling for critical illnesses, as it includes the intermediate state of 'ill'. This state accounts for the fact that an insured individual may remain in this condition for an extended period before death occurs. Such an extension of the traditional two-state model results in a more precise calculation of premiums.

At the same time, the ICIM model is a 'narrowing' of the classical four-state model for critical illness cover, where state (4) represents 'Death from other causes'. Since the probabilities of transitioning to state (4) do not affect the transitions between states in the ICIM model, ICIM proves to be the most suitable for modelling the 'critical illness rider' product.

This model will also be used to analyse the impact of various parameters on premium calculation. As a result, in addition to determining the standard annual net premium for the incurable critical illness rider, we aim to address our research questions as well.

3.2. Data analyses

The data used in this study were obtained from an unnamed insurance company currently operating in Slovakia, which is a subsidiary of a foreign insurer. The dataset comprised information on the number of insured individuals in the insurance portfolio by age (as relevant to the research), gender, health status, survival, and death. The annual data captured the number of individuals at the start and end of each year for specific critical illnesses (though not all), as well as any disabilities caused by these illnesses. The dataset also included specific real-world probabilities of transitioning from a 'healthy' state to a 'critically ill' state and from a 'critically ill' state to a 'deceased' state across different age groups. These probabilities were derived from the insurer's historical data, enabling reliable modelling of transitions between health states within the analysed groups. As a result, the data meet all the necessary criteria for the application of a multi-state stochastic model.

Based on the insurance company's data, we can conclude that the probabilities of contracting a critical illness for the age groups 50 – 64 years are very low. A major role in this is played by early diagnosis, which has improved over the last decade due to increased awareness and the need for preventive examinations. On the other hand, the annual probabilities of remaining in a state of critical illness are relatively high. This is due to medical advances in the treatment of critical diseases, as well as improved care for patients.

Table 1 Annual transition probabilities

Age x	p_x^{11}	p_x^{12}	p_x^{13}	$p_x^{11}+p_x^{12}+p_x^{13}$	p_x^{22}	p_x^{23}	$p_x^{22}+p_x^{23}$
50	0,997237	0,000723	0,002040	1	0,871638	0,128362	1
51	0,996966	0,000802	0,002232	1	0,867415	0,132585	1
52	0,996674	0,000896	0,002430	1	0,859097	0,140903	1
53	0,996329	0,000987	0,002684	1	0,851596	0,148404	1
54	0,995964	0,001100	0,002936	1	0,847211	0,152789	1
55	0,995581	0,001216	0,003203	1	0,843891	0,156109	1
56	0,995191	0,001350	0,003459	1	0,839551	0,160449	1
57	0,994718	0,001499	0,003783	1	0,836322	0,163678	1
58	0,994201	0,001641	0,004158	1	0,833285	0,166715	1
59	0,993606	0,001815	0,004579	1	0,826912	0,173088	1
60	0,992940	0,002034	0,005026	1	0,824793	0,175207	1
62	0,992239	0,002226	0,005535	1	0,823071	0,176929	1
63	0,991568	0,002428	0,006004	1	0,820843	0,179157	1
64	0,990849	0,002681	0,006470	1	0,816619	0,183381	1

Source: The authors processing based on specific data from an unnamed insurance company

3.3. Construction of the Three-state Model

For our purposes, we will extend the two-state 'Healthy - Dead' model to include an additional state: critically ill. This introduces the possibility of recovery, thus a reverse transition from the 'Sick' state back to the 'Healthy' state. We will assume the illness is due to an incurable critical illness (ICI), thereby limiting the mutual transition between the 'Healthy' and 'Sick' states, with no reverse transition possible (see Figure 1). This model, for ease of reference in the article, will be called the Incurable Critical Illness Model (ICIM). The three-state ICIM model is a 'narrowing' of the four-state model for critical illnesses covers, in which state (4) is 'Death from other causes' (Haberman & Pitacco, 1998).

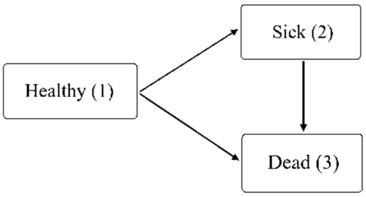

Figure 1 Three-state Model of Incurable Critical Illness
Source: The authors

However, as the article will focus on the 'critical illness rider' product, the probabilities of transition to state (4) will not impact this product.

We consider a time-inhomogeneous Markov chain $\{S(x); 0 \le x < \infty\}$ with a finite state space, where $S(x)$ we denote the state of the process at time *x*. In our case $S(x) \in \{1, 2, 3\}$.

The model has three states:
(1) – Healthy,
(2) – Sick (ill with an incurable critical illness),
(3) – Death.

Generally, when considering a finite number of states (*n*), the symbol ${}_t p_x^{ab}$ denotes the conditional probability

$${}_t p_x^{ab} = P\left[S(x+t) = b \middle| S(x) = a\right]; a, b \in \{1, 2, 3, \ldots, n\}$$

that is, the probability that the process was in state (*b*) at time *x* + *t* given that it was in state (*a*) at time *x*. This stochastic process can be described as Markovian – we assume that the transition probabilities are not influenced by information about the state of the process prior to time *x*.

For state (*k*) and given probabilities, the following properties apply:

- state (*k*) is absorbing, meaning that once the process enters state (*k*), it remains there indefinitely. For all ages *x* and time *t*, it holds that:

$${}_t p_x^{kk} = 1 \text{ and } {}_t p_x^{kb} = 0 \text{ for all } k \neq b \qquad (1)$$

- for all ages *x*, time *t*, and fixed *a*, it applies that:

$$\sum_{b=1}^{n} {}_t p_x^{ab} = 1 \qquad (2)$$

The stochastic processes described by our ICIM model can be understood as continuous Markov processes. Therefore, individual transitions between states can be characterized by transition intensities μ_{x+t}^{ab}, with the following conditions:

$${}_t p_x^{ab} = e^{-\int_0^t \mu_{x+s}^{ab} \, ds} \qquad (3)$$

To calculate individual probabilities, we can use differential equations, as mentioned in the books by Dicskon et al. (2009) and Škrovánková & Simonka (2021). For further actuarial calculations for the given product, we will consider time *t* in years, thus treating transition probabilities as deterministic data.

According to the directed edges (transitions) between nodes (states) in our ICIM model (Figure 1), we can denote the transition probabilities as follows:

${}_t p_x^{11}$ – the probability that a person who was healthy at age x remains healthy at age $x+t$

${}_t p_x^{12}$ – the probability that a person who was healthy at age x becomes ill by age $x+t$

${}_t p_x^{13}$ – the probability that a person who was healthy at age x dies by age $x+t$

${}_t p_x^{22}$ – the probability that a person who was ill at age x remains ill by age $x+t$

${}_t p_x^{23}$ – the probability that a person who was ill at age x dies by age $x+t$

Given the states and transitions between them (Figure 1), the following holds:

$${}_t p_x^{21} = 0, \; {}_t p_x^{32} = 0 \text{ and } {}_t p_x^{33} = 1$$

Based on property (2) for time *t* in our model, we obtain:

$${}_t p_x^{11} + {}_t p_x^{12} + {}_t p_x^{13} = 1 \text{ and } {}_t p_x^{22} + {}_t p_x^{23} = 1$$

As we discussed in the earlier parts of the article, it is crucial for an insurance company to set appropriate premiums for a specific product.

We will focus on calculating the annual net premium P_x. The calculations will be based on real data provided by the insurance company (Table 1), ensuring that the results accurately reflect the relevant circumstances derived from the provided data.

3.4. Application of ICIM model

We will apply our ICIM model to create critical illness rider. This involves a rider for incurable critical illness (ICI), under which the insurer will pay the insured person a regular annual advance payment of €10,000 as long as they remain in this state and do not die. The price for this product is the standard annual premium set by the insurer, paid by potential clients from the inception of the insurance contract while they are in a healthy state. The rider will only be offered to individuals from the age of 18, and the maximum duration of the insurance is until the age of 65.
Note: Beyond this age, a potential client becomes a risk, i.e., the probability of contracting a critical illness is too high for the insurer.

To calculate the annual net premium, we will use the following formula:

$$P_x = \frac{\sum_{t=0}^{64-x} D \cdot {}_t p_x^{11} \cdot p_{x+t}^{12} \cdot v^{t+1} \cdot \ddot{a}^{22}_{x+t+1,\overline{64-(x+t)}|}}{\ddot{a}^{11}_{x,\overline{64-x}|}} \quad (4)$$

where

D – advance annual payment (in our case €10,000)

${}_t p_x^{11}$ – probability of an x-year-old person remaining in state (1) 'healthy' until age $x+t$

p_{x+t}^{12} – annual probability of an $x+t$-year-old healthy person transitioning to state (2) 'Sick' at age $x+t+1$

v – discount factor $v=\frac{1}{1+i}$, where i is the interest rate (in our case 0.7%)

$\ddot{a}^{22}_{x+t+1,\overline{64-(x+t)}|}$ – temporary annuity-due $x+t+1$-year-old person with a term of $64-(x+t)$ years with probabilities ${}_t p_x^{22}$ of remaining in state (2) 'Sick'

$\ddot{a}^{11}_{x,\overline{64-x}|}$ – temporary annuity-due x-year-old person with a term of $64-x$ years with probabilities ${}_t p_x^{11}$ of remaining in state (1) 'Healthy'.

These annuities should be calculated according to formulas mentioned in the books by Bowers et al. (1997) and Krčová et al. (2022).

We will consider a 50-year-old person, calculate the amount of the standard net premium for them, and in the next part, analyse the impact of changes in some probabilities on its amount. It is assumed that the life insurance company has the following annual transition probabilities (for remaining in the same state and transitioning to another state) for ages 50 to 64.

To calculate the standard net premium for a 50-year-old person, probabilities of remaining in the state will also be needed. These are calculated for all ages x and time $t \geq 0$ using annual probabilities (Gerber, 1997)

$${}_t p_x^{ab} = p_x^{ab} \cdot p_{x+1}^{ab} \cdot p_{x+2}^{ab} \cdot \ldots \cdot p_{x+t-1}^{ab} \quad (5)$$

After substituting all the values into equation (4), we obtain the amount of the standard annual net premium for a client today aged 50 years

$$P_{50} = €52.19 \quad (6)$$

The calculated premium is not high in relation to the insurance benefits (payments) paid out. If we consider that the transition annual probabilities from a healthy to a sick state (see Table 1) are very small, it is clear that such a situation is not a great risk for the insurer.

3.5. Analysis of the impact of input parameters on the premium calculation

In creating and pricing its new product, the insurance company closely analyses the impact of input parameters on the premium amount in an effort to maintain its stability, solvency, and profitability.

From equation (4) it is evident that the amount of the premium is influenced not only by the transition probabilities but also by the actuarial interest rate. We will not explore the impact of its changes in this article; instead, we will focus on the effects of changing probabilities.

We will consider several scenarios and analyse the impact of changes in the annual probabilities of transition from a 'Healthy' state to a 'Sick' state, and the probabilities of remaining in the 'Sick' state, on the amount of the standard net premium.

The standard net premium (6) will serve as the reference premium. Changes in the annual probability of transition from the 'Healthy' to the 'Sick' state, whether an increase (+) or a decrease (–) in percentages, with the annual probability of remaining in the 'Sick' state unchanged, as well as the amount of the annual net premium are presented in Table 2.

Table 2 The impact of changing the annual probability of transition to the state 'Sick' on the net premium amount for a 50-year-old person

Scenario	Rate of change p_x^{12} (%)	P_{50}
1	- 20%	€41.75
2	-10%	€46.97
3	0%	€52.19
4	10%	€57.41
5	20%	€62.63

Source: The authors

A reduction in this probability indicates that potential clients are healthier than those previously recorded in the insurance company's data, and vice versa. According to scenarios 1 and 2 in Table 2, clients would pay less annually than the original premium. From the insurance company's perspective, this is advantageous and contributes to increased profitability of the product. For example, with a 10% reduction in the annual probability of becoming 'ill', clients would only pay 90% of the original premium. Conversely, if health conditions deteriorate (as in scenarios 4 and 5 of Table 2), the original premium would be lower than necessary, potentially resulting in losses from the product's sale.

Further analysis of the product will consider the change in the annual probability of remaining in the 'Sick' state, while the annual probability of transition from the 'Healthy' to the 'Sick' state remains unchanged. The results are presented in Table 3.

Table 3 The impact of changing the annual probability of remaining in the state 'Sick' on the net premium amount for a 50-year-old person

Scenario	Rate of change p_x^{22} (%)	P_{50}
1	- 20%	€35.33
2	- 10%	€42.29
3	0%	€52.19
4	10%	€66.82
5	20%	€89.24

Source: The authors

A decrease (increase) in this probability indicates that critically ill clients die quicker (slower) than reflected in the insurance company's data. From the various scenarios, it is evident how the price of the product would change relative to its reference value. For instance, with a 10% reduction in the annual probability of remaining in the 'Sick' state, clients would only need to pay 81.03% of the original premium, thus allowing the insurance company to offer the product cheaper and potentially increase its competitiveness if needed. With a 20% reduction in the annual probability, clients would pay only 67.70% of the original premium, whereas a 20% increase would result in a 20% higher payment.

As the lower annual probability of remaining in the 'Ill' state means that funds from unclaimed benefits stay with the insurer, the product becomes more profitable. Conversely, if improvements in medical care and advancements in medical science led to a statistically higher annual probability of remaining in the 'Sick' state, i.e., a 'beneficiary', the insurer would need to find additional financial resources from its own funds to cover benefit payments. This increases the risk of 'loss' from selling this product.

Let's consider four additional possible scenarios (Table 4). We will contemplate changes in the annual probability of transition from a 'Healthy' to a 'Sick' state (e.g., due to deteriorated or improved living conditions) and changes in the annual probability of remaining in the 'Sick' state (e.g., due to worsening or improving healthcare depending on the country where the client resides).

Table 4 The impact of changes in both annual probabilities p_x^{12} and p_x^{22} on the net premium amount for a 50-year-old person

Scenario	Rate of change p_x^{12} (%)	Rate of change p_x^{22} (%)	P_{50}	ΔP_{50} (%)
1	- 10%	-10%	€38.06	-40,83%
2	10%	-10%	€46.52	-26,99%
3	0%	0%	€52.19	0%
4	- 10%	10%	€60.13	10,86%
5	10%	10%	€73.50	15,21%

Source: The authors

From Table 4, it is clear that changes in both annual probabilities have a significant impact on the change in the amount of the reference premium. Scenario 1 is considered the best case from the insurer's perspective. The insurer could thus save financial resources, invest them appropriately, and distribute a portion of the return to policyholders, for example, in the form of an additional share of the profit. Conversely, scenario 5 could be detrimental to the insurer. Therefore, the insurer should be cautious when entering into insurance contracts, and clients should at least complete a questionnaire that includes questions related not only to health but also to profession and individual lifestyle.

3.6. Discussion

The results of our study indicate that the implementation of the three-state model (ICIM) for calculating critical illness insurance premiums offers significant benefits to the insurance sector, particularly in the development of products with riders for terminal critical illnesses. Modelling for specific age groups suggests that this approach provides more accurate risk estimates, enabling more effective premium determination.

In contrast to the traditional two-state 'Healthy-Dead' model, which does not account for the 'ill' state, the ICIM incorporates this intermediate state, providing more accurate risk estimates related to incurable critical illnesses. Furthermore, compared to the more complex four-state model, which distinguishes between deaths from critical illnesses and deaths from other causes, the ICIM offers a simpler yet more precise interpretation of financial risk for insurers. Differentiating between causes of death does not add significant value to premium calculation when the primary focus is on insuring risks associated with long-term critical illnesses. The ICIM focuses solely on this key factor, simplifying the model without sacrificing accuracy, while enabling insurers to manage claims costs more effectively. As a result, the ICIM strikes a balance between simplicity and accuracy, delivering better outcomes in premium calculation for products targeting incurable critical illnesses.

Our qualitative data analysis revealed that in the 50-64 age group, the probability of developing a critical illness is relatively low, which can be attributed to early diagnosis and greater awareness of preventive check-ups. Conversely, the likelihood of remaining in a critical illness state is relatively high due to advances in medical care and treatment for individuals with critical illnesses.

The impact of model parameters on premium calculation was explored in the quantitative analysis, where we identified several key factors. For the ICIM model, the annual probability of transition from the “Healthy” to the “Sick” state and the probability of remaining in the “Sick” state are of fundamental importance. Our scenario-based simulations demonstrate that a reduction in the probability of transitioning to the “Sick” state lowers the premium, contributing to the competitiveness of the insurance product. However, an increase in this probability may lead to losses for the insurer, highlighting the need for careful monitoring and adjustment of insurance products.

Additionally, we examined the effect of changes in the probability of remaining in the “Sick” state. If this probability decreases, for example, due to a deterioration in the insured’s health, the premium will decrease, allowing the insurer to achieve higher profitability. On the other hand, an increase in this probability, for instance, due to medical advances, could lead to a higher financial burden for the insurer, as claims payouts would become more frequent and extended.

3.6. Results and limitations

Based on data obtained from an unnamed insurance company operating in Slovakia, we applied the ICIM (Incurable Critical Illness Model) to calculate the pure premium for a rider product covering "critical illnesses – incurable critical illnesses." In this paper, we present the results of scenario-based simulations and the calculation of the annual pure premium for a client aged 50 today. To calculate the annual pure premium, we used annual transition probabilities and a discount factor of 0.7%. The resulting pure annual premium for a 50-year-old policyholder was calculated at €52.19, which served as the reference value for this age group in our subsequent analysis.

The analysis of the impact of changes in input parameters, specifically the annual probabilities of transitioning from the "Healthy" to the "Sick" state and the probabilities of remaining in the "Sick" state, illustrates how corresponding percentage decreases or increases in these probabilities affect the final pure annual premium. For example, a 10% reduction in the probability of remaining in the "ill" state leads to a decrease in the pure premium to €42.29, representing a reduction of 23.41% compared to the reference value.

The final analysis evaluates scenarios where both probabilities – transitioning to the "Sick" state and remaining in the "Sick" state – change simultaneously. These simulations highlighted the significant impact of changes in clients' health status on the stability and profitability of the product for the insurer. In the most optimistic scenario from the insurer’s perspective (Table 4 – Scenario 1), the insurer would be able to achieve considerable savings and enhance the competitiveness of its product.

The study also identified several limitations, including reliance on historical data, which may not fully reflect future trends and changes in health outcomes. Furthermore, the models used may not entirely capture the rapid advancements in medical science or shifts in demographic trends, both of which can significantly impact the incidence and progression of critical illnesses. As insurance markets and the associated risks constantly evolve, it is crucial to regularly analyse and update the parameters of insurance event models. This also involves monitoring and processing a large volume of new data, from the perspective of the insurance company at the Not Small – Not Big Data (NoS-NoB, defined by Schmidt (2024)) level, revising models, and adapting them according to the internal requirements of the insurance company.

Conclusion

The article demonstrated the advantages of using stochastic models in managing insurance products. By extending the traditional 'Healthy-Dead' model to a three-state 'Healthy-Sick-Dead' model, where the illness is due to an incurable critical illness, we introduced the Incurable Critical Illness Model (ICIM). Using the ICIM, we were able to more accurately model and predict the risks associated with incurable critical illnesses. This approach enables insurance companies to set supplementary insurance valuations more effectively. It more precisely reflects actual risks, thereby contributing to the financial stability and solvency of the insurance company in accordance with European Union standards and regulations.

The results of the analysis showed that changes in transition probabilities have a significant impact on the net premium. For instance, reducing the probability of transitioning from the 'Healthy' to the 'Sick' state leads to a lower premium, which can allow the insurance company to offer more competitively priced products. Conversely, improvements in healthcare, which increase the probability of remaining in the 'Sick' state, can lead to higher expenses for the insurance company, highlighting the need for careful estimation of these parameters.

Moreover, the study revealed that changes in these transition probabilities can significantly affect the financial stability of insurers. Adjustments in health status transitions can either reduce costs and increase product competitiveness or raise expenses, emphasising the importance of regular model updates to reflect evolving health conditions. Accurate estimation of these probabilities is necessary for maintaining solvency and financial stability.

While the ICIM model offers significant advantages in modelling critical illnesses and calculating premiums, several limitations need to be considered. The model's reliance on historical data implies that the accuracy of the results could be affected by the availability and quality of these data. Moreover, although the three-state model is effective for analysing transitions between health states, it does not take into account additional factors, such as advancements in healthcare technology, which could substantially affect the expected duration of critical illness.

In the future, extending this model by incorporating additional parameters or adapting it to account for evolving health conditions would allow for more accurate premium calculations. Such improvements could enhance the financial stability of insurers while ensuring more equitable insurance terms for clients.

In conclusion, we can state that stochastic models represent a powerful tool for optimising the processes of managing insurance products and supplementary insurances, leading to a better understanding of risks and more efficient risk valuation arising from insurance contracts. This is key for insurance product management, long-term success, and the stability of insurance institutions.

Declarations Availability of data and materials

The data used and analysed in this article were provided exclusively for the purposes of this research. The dataset is part of the internal data from an unnamed (but real) insurance company, and its availability is governed and restricted by the company's strict internal policies. The data are available only to the extent that they have been processed, as stated by the authors of the article.

Funding

This work has been supported by the projects VEGA 1/0410/22 - Analyses of insurance risks in relation to managing a life insurance company and VEGA 1/0096/23 - Selected methods of risk management in the implementation of partial internal models for determining the capital requirement for solvency.

Acknowledgements

Not applicable.

References

Alim, K., Listiani, A., Anggraeni, A. S., & Efendie, A. R. (2019). Critical illness insurance pricing with stochastic interest rates model. *Journal of Physics: Conference Series, 1341*(1), 062026. https://doi.org/10.1088/1742-6596/1341/6/062026

Andersen, P. K., & Ravn, H. (2023). Intensity models. *Models for Multi-State Survival Data*, 69–116. https://doi.org/10.1201/9780429029684-3

Anderson, R. M., & May, R. M. (2020). *Infectious Diseases of Humans: Dynamics and Control*. Oxford University Press.

Biener, C. (2019). *Usage-Based Insurance in the Automobile Insurance Market: Empirical Analyses and Design of Regulatory Approaches*. Springer.

Bowers, N. L., Gerber, H. U., Hickman, J. C., Jones, D. A., & Nesbitt, C. J. (1997). *Actuarial Mathematics* (2nd ed.). Society of Actuaries.

Brokešová, Z., Ondruška, T., & Pastoráková, E. (2023). Determinants of personal risk financing strategies. Journal of Economics, 71(10), 579–595. https://doi.org/10.31577/ekoncas.2023.10.01

Campbell, J. Y., Lo, A. W., & MacKinlay, A. C. (1997). *The Econometrics of Financial Markets*. Princeton University Press.

Cerqueti, R., D'Ecclesia, R. L., & Levantesi, S. (2021). Preface: Recent developments in financial modelling and risk management. *Annals of Operations Research, 299*(1), 1–5. https://doi.org/10.1007/s10479-021-03958-9

Choulli, T., Daveloose, C., & Vanmaele, M. (2021). Mortality/Longevity Risk-Minimization with or without Securitization. *Mathematics*, *9*(14), 1629. https://doi.org/10.3390/math9141629

Christiansen, M. C., & Niemeyer, A. (2015). On the forward rate concept in multi-state life insurance. *Finance and Stochastics, 19(2)*, 295–327. https://doi.org/10.1007/s00780-014-0244-9

Clemente, C., Guerreiro, G. R., & Bravo, J. M. (2023). Modelling motor insurance claim frequency and severity using gradient boosting. *Risks, 11*(163). https://doi.org/10.3390/risks11090163

Colaneri, K., Cretarola, A., & Salterini, B. (2024). Optimal investment and reinsurance under exponential forward preferences. *Mathematics and Financial Economics.* https://doi.org/10.1007/s11579-024-00372-0

D'Amato, V., Levantesi, S., & Menzietti, M. (2020). De-risking long-term care insurance. *Soft Computing, 24*(12), 8627–8641. https://doi.org/10.1007/s00500-020-04823-9

D'Amico, G., Guillen, M., Manca, R., & Petroni, F. (2017). Multi-state models for evaluating conversion options in life insurance. *Modern Stochastics: Theory and Applications, 4*(1), 1-13. https://doi.org/10.15559/17-VMSTA78

Dickson, D. C. M., Hardy, M. R., & Waters, H. R. (2009). *Actuarial Mathematics for Life Contingent Risks*. Cambridge University Press. https://doi.org/10.1017/CBO9780511845242

Fu, M. C., & Hu, J.-Q. (2002). *Stochastic Processes and Models in Operations Research*. Springer.

Garmendia Bergés, L., Cortés Martínez, J., Gómez Melis, G., et al. (2023). MSMpred: Interactive modelling and prediction of individual evolution via multistate models. *BMC Medical Research Methodology, 23*, 126. https://doi.org/10.1186/s12874-023-01951-3

Gautier, G. (2016). *Stochastic Modeling of Geological Structures*. Elsevier. https://doi.org/10.1016/B978-0-12-801490-8.00001-8

Gerber, H. U. (1997). *Life Insurance Mathematics* (3rd ed.). Springer. https://doi.org/10.1007/978-3-662-03449-2

He, Y. (2022). Stochastic processes with applications in physics and insurance. *Bulletin of the Australian Mathematical Society, 106*(3), 513–517. https://doi.org/10.1017/S000497272200079X

Hellriegel, J. (2019). *Insurance Product Management: Manually Underwrite, Rate, and Compete in the Digital World*. Actuarial Bookstore.

Hipel, K. W. (2000). *Stochastic and Statistical Methods in Hydrology and Environmental Engineering: Effective Environmental Management for Sustainable Development*. Springer. https://doi.org/10.1007/978-94-009-1978-3

Hu, R., & Laurière, M. (2023). Recent Developments in Machine Learning Methods for Stochastic Control and Games. *Journal of Applied Mathematics, 45*(3), 123-145. https://doi.org/10.1007/s10462-022-10123-4

Ishikawa, M. (2022). On the mathematical analysis for the stochastic infectious model under subclinical infections and vaccination. *Proceedings of the ISCIE International Symposium on Stochastic Systems Theory and Its Applications, 2022*, 1–7. https://doi.org/10.5687/sss.2022.1

Itô, K. (1944). Stochastic integral. *Proceedings of the Imperial Academy, Tokyo*, 20(8), 519-524. https://doi.org/10.3792/pia/1195572786

Kolmogorov, A. N. (1940). *Teoriya veroyatnostei*. Gosudarstv. izdavatelstvo fiziko-matematicheskoi literatury.

Krčová, I., Sakálová, K., & Zelinová, S. (2022). *Moderné metódy určovania poistného v životnom poistení* (1st ed.). H.R.G.

Levantesi, S., Menzietti, M., & Nyegaard, A. K. (2024). *De-risking in multi-state life and health insurance*. *Annals of Actuarial Science, 18*(2), 423 - 441. https://doi.org/10.1017/S1748499524000083

McNeil, A. J., Frey, R., & Embrechts, P. (2015). *Risk Management: Principles and Practice* (2nd ed.). Princeton University Press. https://doi.org/10.1515/9781400854780

Mišút, M. (2021). Posudzovanie pripravenosti podniku na nasadenie veľdát (big data). *Ekonomika a informatika, 19* (1). SSHI. https://ei.fhi.sk/index.php/EAI/article/view/219

Mohammadi, H., Marateb, H. R., Momenzadeh, M., Wolkewitz, M., & Rubio-Rivas, M. (2024). Tracing in-hospital COVID-19 outcomes: A multistate model exploration (TRACE). *Life, 14*(9), 1195. https://doi.org/10.3390/life14091195

Mozokhina, A., Popravka, I., Saade, M., & Volpert, V. (2024). Modeling the influence of lockdown on epidemic progression and economy. *Mathematics, 12*(19), 3106. https://doi.org/10.3390/math12193106

Neuman, S. P., & Wierenga, P. J. (2012). *Stochastic Methods in Subsurface Contaminant Hydrology*. Dover Publications.

Palmer, T. (1999). *Stochastic Climate Models*. Cambridge University Press. https://doi.org/10.1017/CBO9780511581348

Páleš, M. (2019). Data quality and its importance - actuarial view. *Slovenská štatistika a demografia*, 29(1). Štatistický úrad SR. https://ssad.statistics.sk/SSaD/index.php/data-quality-a-its-importance-actuarial-view/?lang=en

Poláček, Š., & Páleš, M. (2012). Risk management interest rate changes and duration in insurance companies. *Managing and modelling of financial risks : 6th international scientific conference*. Ostrava - Poruba: Vysoká škola báňská - Technická univerzita Ostrava

Rogers, E. M. (2003). *Diffusion of Innovations* (5th ed.). Free Press.

Rogers, T. (2011). *Stochastic Models in Biology*. Springer. https://doi.org/10.1007/978-1-4419-7046-6

Schmidt, P. (2024). *Not Small - Not Big Data (NoS-NoB) v kontexte dátovej vedy: monografia*. Nové Zámky: 5V, o.z.

Schmidt, P. (2021). Dátová veda a technológia veľdát v službách aktuárov. *Ekonomika a informatika* 19 (1). SSHI. https://ei.fhi.sk/index.php/EAI/article/view/236

Shen, W. (2024). Optimal investment and reinsurance strategies for an insurer with regime-switching. Mathematics and Financial Economics, 18(4), 555–576. https://doi.org/10.1007/s11579-024-00374-y

Shreve, S. E. (2004). *Stochastic Calculus for Finance I: The Binomial Asset Pricing Model*. Springer. https://doi.org/10.1007/978-0-387-27204-6

Simpson, D. J. W., Feudel, U., & Siekmann, J. (Eds.). (2012). *Stochastic Processes and Applications in Biology and Medicine II: Theory, Methods, and Models*. Springer. https://doi.org/10.1007/978-3-642-19566-4

Škrovánková, L., & Simonka, Zs. (2021). *Aktuárske metódy a modely v penzijnom, zdravotnom a nemocenskom poistení*. H.R.G.

Tadayon, V., & Ghanbarzadeh, M. (2024). Spatial copula-based modeling of claim frequency and claim size in third-party car insurance: A Poisson-mixed approach for predictive analysis. *Insurance: Mathematics and Economics, 119*, 119–129. https://doi.org/10.1016/j.insmatheco.2024.08.005

Weeks, E. P. (2010). *Stochastic Weather Forecasting*. Wiley.

Wilkinson, D. J. (2006). *Stochastic Modelling for Systems Biology*. Chapman & Hall/CRC. https://doi.org/10.1201/9781420011913

Zhao, Y., Wang, H., & Wang, D. (2024). The dynamic behavior of a stochastic SEIRM model of COVID-19 with standard incidence rate. *Mathematics, 12*(19), 2966. https://doi.org/10.3390/math12192966

Zuhairoh, F., Rosadi, D., & Efendie, A. R. (2021). Determination of basic reproduction numbers using transition intensities multi-state SIRD model for COVID-19 in Indonesia. *Journal of Physics: Conference Series, 1821*(1), 012050. https://doi.org/10.1088/1742-6596/1821/1/012050

Zuhairoh, F., Rosadi, D., & Effendie, A. R. (2024). Multi-state SVIRD model with continuous-time Markov chain assumption on the spread of infectious diseases. *Austrian Journal of Statistics, 53*(1), 70–94. https://doi.org/10.17713/ajs.v53i1.1633

✉ **Correspondence**

Zsolt Simonka

University of Economics in Bratislava
Department of Mathematics and Actuarial Science
Dolnozemská cesta 1/b, 852 35, Bratislava, Slovak Republic
E-mail: zsolt.simonka@euba.sk