

Managerial conflict of interests effects on duopoly market structure

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Abstract

Background: Principal-agent problem has been discussed intensively in the recent decade, but this specific case has not yet been analyzed in this manner; one of two companies in Cournot duopoly employs a manager who has a partial ownership of the second company, but without executive power in that company. This kind of conflict of interest changes the market game since the overlapping makes it difficult to determine how many actual players there are on the market, which is crucial to understanding what is about to happen with prices and quantities.

Purpose: This paper will determine which agent's share in the other company becomes a problem for the principal of the first company and how a change in the share affects market price, both companies' quantities and profits, and finally how a duopoly grows closer to a monopoly since the number of players is no longer integer.

Study design/methodology/approach: The manager of the first company is paid in that company's share in profit. As a partial owner of the other company, this manager also receives ownership revenue. Thus the manager (agent) tries to maximize his own revenue which consists of the share in both companies. The agent's actions in the first company are aimed to maximize his own profit instead of the principal's profit.

Findings/conclusions: The higher the agent's share in the competitive company, the greater the agent's reward has to be in terms of the share in the profit of the first company. Additionally, it also increases the prices, decreases the quantities, turning duopoly into a non-integer oligopoly, the closer to monopoly the higher the agent's share is in the competitive company.

Limitations/future research: The assumed Cournot game should also be transformed into a game where players do not act simultaneously. Therefore, a Stackelberg oligopoly analysis could bring a novel view of this specific interaction.

Keywords

Principal-agent problem, conflict of interest, Cournot duopoly, non-integer number of players, monopoly

Introduction

According to the agency theory, one party (the principal) delegates work to another party (the agent) (Daily et al., 2003; Shapiro, 2005; Bosse & Phillips, 2016). The application of the theory has been widespread, including economics (Cooper, 1949, 1951; Ross, 1973), management (Barnard, 1938; Eisenhardt, 1985, 1988; Kosnik &

Bettenhausen, 1992, Zhang et al., 2022; Matinheikki, 2022), finance (Jensen & Meckling, 1976, Fama, 1980, Forster et al., 2025), politics (Mitnick, 1982, 1990; Hammond & Knott, 1996, Kiser & Tong, 1992, Al-Faryan, 2024), and sociology (Eccles, 1985; White, 1985; Shapiro, 1987, Davis et al., 2021). The principal-agent problem refers to the conflict in interests that arises when one party (the agent) takes actions on behalf

of another party (the principal) (Eisenhardt, 1985). In practice, agency problem arises when the agent (e.g. manager) fails to act in the interest of the principal (e.g. owner) (Williamson, 1975; Arrow, 1984). Berle and Means (1932) conclude that the enforcement of the corporate law in the early 1930s in the U.S. allowed managers to be able to manage the resources of companies to their own advantage. Fama and Jensen, (1983) show that separation of ownership and control lead to the information asymmetry.

The purpose of this paper is to provide a mathematical model for conflicts of interest in a Cournot type duopoly, in which the principal-agent problem has quantitative effects both on the principal's agent and co-owners. After the introduction, the first part of the paper presents the theoretical background of the principal-agent conflict of interest. The second part presents a model which introduces the agency problem in canonical duopoly. It is followed with the Results and Discussion section which employs the comparative static analysis of the market quantity, price and companies' profit reaction to the change in the agent's share in competitive company, with algebraic analysis and numerical simulations. Conclusion summarizes the main findings and sheds a light on the possible future studies.

1. Theoretical background

The paper offers the extension of the model initially presented in Vrankić et al. (2022). The theoretical background of the principal-agent conflict of interest has been well-rounded and little break-through came up since the turn of the century (Vrankić et al., 2022).

Jensen and Meckling (1976) define an agency relationship as a contract between one or more parties (principals) and another party (agents), entrusting them with the responsibility of allocating resources on their behalf. If both parties are motivated by maximizing their own wealth, there is good reason to believe that agents will not always act in the best interests of the principal. As a result, the agency problem arises when the interests of the principal and agent conflict, and/or when the principal finds it difficult to verify the actions of the agent (Eisenhardt, 1989). In this situation, the agent is free to pursue his/her own interests in order to benefit from its position (Noreen, 1988; Cohen et. al, 2007). It is also possible for agents to exploit information asymmetry to take (hidden) actions in order to benefit themselves at the expense of the principal

(Holmstrom, 1979; Panda & Leepsa, 2017). In order to prevent divergence of interests, the principal may establish an appropriate incentive program (Jensen, 1994; Laffont & Martimort, 2009) or incur oversight costs in order to prevent unwanted agent behavior (Donaldson and Davis, 1991; Bonazzi & Islam, 2007). Accordingly, it is impossible for the agent to make optimal decisions without incurring costs (both for the agent and the principal). According to Jensen and Meckling (1976), agency costs include monitoring costs, bonding costs, and residual losses.

It is argued that the agency theory fails to capture both sides of the relationship (i.e. the relationship between the principal and the agent), therefore failing to provide insight into the potential problem of the principal exploiting the agents (Shapiro, 2008). A key component of Perrow's stewardship theory is the rejection of the assumption that agents are work-averse, self-interested utility maximizers. In spite of this, he acknowledges that there are certain situations that are more likely to result in the emergence of agency problems.

Two lines of research have been conducted on the agency problem: positivist theory (Jensen & Smith, 2000) and principal-agent theory (Eisenhardt, 1989). In positivist theory, conflicting goals are identified and governance mechanisms are described to limit the agent's selfish behavior; it is less mathematical and focuses almost exclusively on the special case of large, public corporations (Jensen & Meckling, 1976; Fama, 1980; Fama & Jensen, 1983). Principal-agent research provides a rather general understanding of agency relationships (i.e. it impacts a variety of agency relationships such as client-lawyer, writer-publisher, owner-manager, etc.). A logical deduction and mathematical proof are followed by the specification of assumptions (e.g. Demski & Feltham, 1978). By developing a mathematical model for the special case of an agency relationship between the owner of a company and its manager who owns shares in another company in the same industry, this paper attempts to bridge the two streams. It is assumed that the market is Cournot duopoly (non-cooperative companies producing a homogeneous product and bringing their decisions simultaneously). Because it analyzes a special case with a high risk of agency problem occurrence (i.e. the stewardship theory's premise will not hold), it does not contradict the alternative view of principal-agent problem (Donaldson & Davis, 1991). Furthermore, the present research provides

empirical evidence regarding the impact of agency problems on principal wealth (Crutchley & Hansen, 1989, Tosi Jr., & Gomez-Mejia, 1989, Lafontaine, 1992, Davidson III et al., 2004). A limited number of studies have investigated the broader social and institutional effects of agency problems. In the opinion of Hill and Jones (1992), an area that remains relatively unexplored is the capacity of agency theory to explain the nature of the contractual relationships between a firm's stakeholders (e.g. employees, customers, suppliers, creditors, communities, and the general public). In the meantime, this topic "grew cold" and few studies have recently focused on the application of principal-agent problem, but almost none have addressed oligopoly (Chen et al., 2022). It is the intention of this paper to fill this gap in the literature by developing a theoretical model to illustrate how agency theory directly negatively impacts company performance and the wealth of its owners, as well as indirectly negatively affecting consumers and the general public's interests since principal-agent problem in oligopoly is a gamechanger and this paper aims to prove it.

2. Models

One side of the story refers to the agency problem. The other is the plain Cournot duopoly. The overlapping of these two aspects shall provide the grounds for the analyses required to achieve the paper's goals.

2.1. Basic Cournot oligopoly model

In the Cournot duopoly model, each company announces its own profit-maximizing production level based on the production level of its competitor. There is no incentive for two companies to change their production level at the same time in Cournot equilibrium, which is one of the basic characteristics of Nash equilibrium too. Their production plans are announced simultaneously with the production of homogeneous products. Using standardized prices to simplify calculations, and maintain the explanatory power of the model, their joint quantity produced affects the price with the following relationship:

$$p = 1 - Y = 1 - y_1 - y_2 \quad (1)$$

where y_1 and y_2 are nonnegative quantities produced by these two companies. The second simplification, which allowed for price standardization, is that there is no cost assumption,

which does not reduce the model's explanatory power.

$$\begin{aligned} \max_{y_1} \pi_1 &= p y_1 \\ &\& \\ \max_{y_2} \pi_2 &= p y_2 \end{aligned} \quad (2)$$

When maximizing profit ($\frac{\partial \pi_1}{\partial y_1} = 0$ & $\frac{\partial \pi_2}{\partial y_2} = 0$) the following reaction curves are obtained:

$$\begin{aligned} y_1 &= \frac{1}{2} - \frac{1}{2} y_2 \\ &\& \\ y_2 &= \frac{1}{2} - \frac{1}{2} y_1 \end{aligned} \quad (3)$$

Their intercept provides Cournot-Nash equilibrium where quantities account for 1/3 of the perfectly competitive market coverage, 2/3 in total:

$$y_1^C = y_2^C = \frac{1}{3}, Y^C = \frac{2}{3} \quad (4)$$

Price in the basic Cournot model is $p^C = 1 - \frac{2}{3} = \frac{1}{3}$ and profits, which are equal to revenues under the zero cost assumption, are equal to the quantities multiplied with price:

$$\pi_1^C = \pi_2^C = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}, \pi = \frac{2}{9} \quad (5)$$

2.2. Modified Cournot duopoly model

The main idea of this paper is based on a specific situation where the Principal (owner of the Company 1) employs the Agent (minor shareholder of the competitive Company 2, but without executive power in that company) as the manager of Company 1 with full executive power. It is also assumed that he pays the Agent with the share in total profit of the Company 1, ϑ_1 . The Agent owns ϑ_2 part of the Company 2 ($\vartheta_2 < 50\%$, since he has a non-controlling share).

This kind of the overlapping causes a conflict of interests since the Agent's income consists of both labor income (from Company 1, L) and the capital income (from Company 2, R):

$$V = L + R = \vartheta_1 \Pi_1 + \vartheta_2 \Pi_2 \quad (6)$$

Thus, the manager's goal is inconsistent with the owner's goal, which creates an agent-principal problem, where the agent acquires yield from both parties. Depending on the shares of companies' profits, ϑ_1 and ϑ_2 , the Agent's goal would be more or less coherent with the principal's goal, maximizing their own income. Since the Agent has executive power only in the Company 1, as long as $\vartheta_2 < 0.5$, the decision variable remains y_1 only (otherwise, he would achieve executive power in both companies which would instantly lead to creation of a specific kind of a split monopoly, but

not cartel). Therefore, the Agent tries to maximize his own revenues, V :

$$\max_{y_1} V = \vartheta_1 \Pi_1 + \vartheta_2 \Pi_2 \quad (7)$$

$$\max_{y_1} V = \vartheta_1 p y_1 + \vartheta_2 p y_2 \quad (8)$$

$$\max_{y_1} V = p(\vartheta_1 y_1 + \vartheta_2 y_2) \quad (9)$$

$$\frac{\partial V}{\partial y_1} = 0 \quad (10)$$

$$\begin{aligned} \frac{\partial p}{\partial y_1} (\vartheta_1 y_1 + \vartheta_2 y_2) + p \vartheta_1 &= 0 \\ \vartheta_1 - 2\vartheta_1 y_1 - (\vartheta_1 + \vartheta_2) y_2 &= 0 \\ \Rightarrow y_1 &= \frac{1}{2} - \frac{\vartheta_1 + \vartheta_2}{2\vartheta_1} y_2 \end{aligned} \quad (11)$$

where (11) is the Company 1's reaction curve. Note that, since the Agent has all the executive power in Company 1, company 1's reaction curve is deducted from the Agent's profit function, not its own. In contrast, Company 2 makes its own decisions and has its own profit maximizing reaction:

$$\max_{y_2} \Pi_2 = p y_2 = (1 - y_1 - y_2) y_2 \quad (12)$$

$$\frac{\partial \Pi_2}{\partial y_2} = 1 - y_1 - 2y_2 = 0 \quad (13)$$

$$\Rightarrow y_2 = \frac{1}{2} - \frac{1}{2} y_1 \quad (14)$$

where (14) is the Company 2's reaction curve. The intercept of the two reaction curves (11 & 14) provides the market equilibrium in Cournot model with conflict of interest (point C):

$$y_1^{CC} = \frac{\vartheta_1 - \vartheta_2}{3\vartheta_1 - \vartheta_2}, y_2^{CC} = \frac{\vartheta_1}{3\vartheta_1 - \vartheta_2}, Y^{CC} = \frac{2\vartheta_1 - \vartheta_2}{3\vartheta_1 - \vartheta_2} \quad (15)$$

Stubbing (15) into demand provides the market price:

$$p = \frac{\vartheta_1}{3\vartheta_1 - \vartheta_2} \quad (16)$$

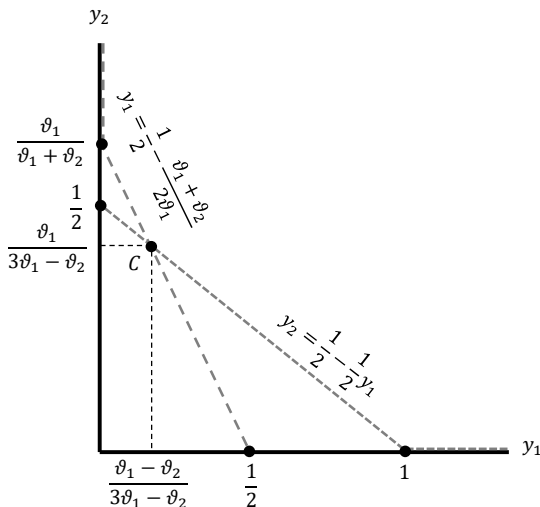


Figure 1 Cournot model in case of Agent's conflict of interests

Source: the authors

Profits of these two companies are:

$$\pi_1 = \frac{1}{9} - \frac{\vartheta_2(3\vartheta_1 + \vartheta_2)}{9(3\vartheta_1 - \vartheta_2)^2} \quad (17)$$

$$\pi_2 = \left[\frac{1}{3} + \frac{\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} \right]^2 \quad (18)$$

$$\pi = \frac{2}{9} + \frac{\vartheta_2(3\vartheta_1 - 2\vartheta_2)}{9(3\vartheta_1 - \vartheta_2)^2} \quad (19)$$

2.3. Relation between Basic and Modified Cournot duopoly model

Market quantities, price and profits in the modified model can be rearranged as follows:

$$y_1^{CC} = \frac{\vartheta_1 - \vartheta_2}{3\vartheta_1 - \vartheta_2} = \frac{1}{3} - \frac{2\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} < \frac{1}{3} = y_1^C \quad (20)$$

$$y_2^{CC} = \frac{\vartheta_1}{3\vartheta_1 - \vartheta_2} = \frac{1}{3} + \frac{\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} > \frac{1}{3} = y_2^C \quad (21)$$

$$Y^{CC} = \frac{2\vartheta_1 - \vartheta_2}{3\vartheta_1 - \vartheta_2} = \frac{2}{3} - \frac{\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} < \frac{2}{3} = Y^C \quad (22)$$

$$p^{CC} = \frac{\vartheta_1}{3\vartheta_1 - \vartheta_2} = \frac{1}{3} + \frac{\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} > \frac{1}{3} = p^C \quad (23)$$

$$\pi_1^{CC} = \frac{1}{9} - \frac{\vartheta_2(3\vartheta_1 + \vartheta_2)}{9(3\vartheta_1 - \vartheta_2)^2} < \frac{1}{9} = \pi_1^C \quad (24)$$

$$\pi_2^{CC} = \left[\frac{1}{3} + \frac{\vartheta_2}{3(3\vartheta_1 - \vartheta_2)} \right]^2 > \frac{1}{9} = \pi_2^C \quad (25)$$

$$\pi^{CC} = \frac{2}{9} + \frac{\vartheta_2(3\vartheta_1 - 2\vartheta_2)}{9(3\vartheta_1 - \vartheta_2)^2} > \frac{2}{9} = \pi^C \quad (26)$$

It is proven that in this model Principal's company 1 produces less and earns less when the Agent has the conflict of interests; competitive company, where the Agent has the share, produces more and earns more; prices soar above the basic Cournot level of prices and the overall profit on the market is also above the overall Cournot profit without conflict of interest, showing a negative effect of the conflict on consumers and competition.

Market saturation in the Cournot oligopoly model can also be analyzed by the number of companies in the oligopolistic market. Fundamental microeconomic theory provides the information that the overall produced quantity on the Cournot oligopoly market (Y^C) with n companies is equal to $Y^C = \frac{n}{n+1} Y^{PC}$, where Y^{PC} is the perfectly competitive quantity. In the example presented in (1), Y^{PC} turns out to be equal to 1. Therefore, each total quantity of production in Table 1 could provide an equivalent of the market participants:

$$Y = \frac{n}{n+1} \Rightarrow n = \frac{Y}{1-Y} \quad (27)$$

It enables numeric comparison between the basic and the conflicted Cournot duopoly for any chosen Company 2 share ϑ_2 (Table 1).

Table 1 Conflict of interests and the equivalent number of market players

ϑ_2	0	.05	.25	.50	.75
Y^{CC}	0.667	0.630	0.590	0.500	0.500
n	2.000	1.703	1.441	1.000	1.000

Source: the authors

Notice that for the values of $\vartheta_2 \geq 0.5$ Company 1 is eliminated and Company 2 becomes a monopolist. Also notice that the quantity produced, which can be expressed through the number of participants on the market, gradually falls as ϑ_2 increases, showing that the larger the Agent's share in Company 2, the lower the equivalent number of participants is, corroborating the previous statements that the rise in the conflict of interests discourages competition. It in turn proves that the real monopolistic power lies in the overlapping ownership which is the cause of the agency problem in this case.

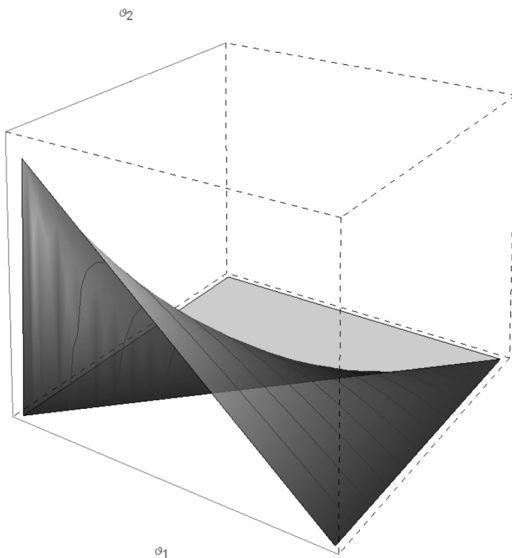
3. Results and Discussion

The owner of Company 1 (Principal) is left with the rest of the profit, Z , after he paid the Agent (manager):

$$\max_{\vartheta_1} Z = (1 - \vartheta_1)\Pi_1 = \frac{(1 - \vartheta_1)\vartheta_1(\vartheta_1 - \vartheta_2)}{(3\vartheta_1 - \vartheta_2)^2} \quad (28)$$

Principal then wants to find out the profit share ϑ_1 which maximizes his own profit for a given level of ϑ_2 he cannot control:

$$\max_{\vartheta_1} Z = \frac{(1 - \vartheta_1)\vartheta_1(\vartheta_1 - \vartheta_2)}{(3\vartheta_1 - \vartheta_2)^2} \quad (29)$$

**Figure 2** Principal's profit function Z
Source: the authors

Remember that $\vartheta_1 \in [0, 1]$ by definition refers to the Figure 1; one notices that if $\frac{\vartheta_1}{\vartheta_1 + \vartheta_2} \leq \frac{1}{2}$ then the

Agent decides not to produce and competitive company 2 becomes a monopolist ($y_2^M = \frac{1}{2}$).

As a result, the Principal must provide the following in order to maintain its position on the market:

$$\frac{\vartheta_1}{\vartheta_1 + \vartheta_2} > \frac{1}{2} \Rightarrow \vartheta_1 > \vartheta_2 \quad (30)$$

Secondly, one has to discuss the closed set $\vartheta_2 \in [0, 1]$ since when $\vartheta_2 = 1$ then it is impossible that share ϑ_1 exceeds the value of ϑ_2 making $Z = 0$. Therefore the optimum share of profit rewarded to the Agent has to be found at $\vartheta_1 \in (\vartheta_2, 1)$, where by definition $\vartheta_2 < \frac{1}{2}$.

$$\frac{dZ}{d\vartheta_1} = \frac{-3\vartheta_1^3 + 3\vartheta_2\vartheta_1^2 + \vartheta_2(1 - 2\vartheta_2)\vartheta_1 + \vartheta_2^2}{(3\vartheta_1 - \vartheta_2)^3} = 0 \quad (31)$$

The expression (31) can be solved as the cubic equation:

$$-3\vartheta_1^3 + 3\vartheta_2\vartheta_1^2 + \vartheta_2(1 - 2\vartheta_2)\vartheta_1 + \vartheta_2^2 = 0 \quad (32)$$

Therefore a solution will be $\vartheta_1^* = \vartheta_1(\vartheta_2)$ and can be obtained by solving a cubic equation for each given value of ϑ_2 . That solution will provide relation between the Agent's ownership of the company 2 and the profit share as a reward from the Principal for managing Company 1. That result will, in turn, provide relation between ϑ_2 and the share in the Company 1's profit, individual production levels, total profits of the companies and the market price. The following sensitivity analyses will provide answers to these questions.

3.1. Effects of ϑ_2 change on Agent's reward

This section will provide the exact algebraic relation between the Agent's share in Company 2 and their profit share in Company 1. The answer to that question lies in solution of (32):

Its differentiation with respect to ϑ_2 gives:

$$\left[9\vartheta_1^{*2} - 6\vartheta_2\vartheta_1^* - \vartheta_2(1 - 2\vartheta_2) \right] \frac{d\vartheta_1^*}{d\vartheta_2} - 3\vartheta_1^{*2} - 2\vartheta_2 - \vartheta_1^*(1 - 4\vartheta_2) = 0 \quad (33)$$

Now $\frac{d\vartheta_1^*}{d\vartheta_2}$ can be obtained, and the resulting fraction simplified as A/B:

$$\frac{d\vartheta_1^*}{d\vartheta_2} = \frac{3\vartheta_1^{*2} + 2\vartheta_2 + \vartheta_1^*(1 - 4\vartheta_2)}{9\vartheta_1^{*2} - 6\vartheta_2\vartheta_1^* - \vartheta_2(1 - 2\vartheta_2)} = \frac{A}{B} \quad (34)$$

The next step is to determine the sign of A. Multiplying A with ϑ_1^* provides:

$$A\vartheta_1^* = 3\vartheta_1^{*3} + 2\vartheta_2\vartheta_1^* + \vartheta_1^{*2}(1 - 4\vartheta_2) \quad (35)$$

Now express $3\vartheta_1^{*3}$ from (24) and replace the bolded part of (27) to obtain:

$$A\vartheta_1^* = (1 - \vartheta_2)\vartheta_1^{*2} + \vartheta_2(3 - 2\vartheta_2)\vartheta_1^* + \vartheta_2^2 > 0 \quad (36)$$

(36) is obviously positive, due to the assumptions about ϑ_2 and the sign of the square. Next step is determination of the B sign:

$$B = 9\vartheta_1^{*2} - 6\vartheta_2\vartheta_1^* - \vartheta_2(1 - 2\vartheta_2) \quad (37)$$

After multiplication of (29) with ϑ_1^* one obtains:

$$B\vartheta_1^* = 9\vartheta_1^{*3}(\vartheta_2) - 6\vartheta_2\vartheta_1^{*2} - \vartheta_2(1 - 2\vartheta_2)\vartheta_1^* \quad (38)$$

Now expressing $\vartheta_2(1 - 2\vartheta_2)\vartheta_1^*$ from (32) and replacing the bolded part of (38):

$$B\vartheta_1^* = 3\vartheta_1^{*2}(2\vartheta_1^* - \vartheta_2) + \vartheta_2^2 > 0 \quad (39)$$

Knowing (30), (39) cannot be anything but positive. By dividing $A\vartheta_1^*$ and $B\vartheta_1^*$ one obtains (40) which is identical to (34):

$$\frac{d\vartheta_1^*}{d\vartheta_2} = \frac{A\vartheta_1^*}{B\vartheta_1^*} = \frac{A}{B} > 0 \quad (40)$$

This finding proves that the Principal's optimal reward for the Agent increases as the Agent's profit share in the competitive company rises, keeping in mind that the rewarded share should be greater than the Agent's share in the competitive company (expression 30).

3.2. Effects of ϑ_2 change on output, profits and price

In Chapter 1.3 the relation between basic and conflicted duopoly for different ϑ_2 is shown, but no sensitivity analysis has been conducted so far. Observing the equations (20) through (26), for the optimal value ϑ_1^* , one notices that variation of the common factor:

$$f(\vartheta_2) = \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} = D \quad (41)$$

is the only factor that affects the change in all but π_2^{CC} and π^{CC} . Therefore first step is to determine dynamics of (41):

$$f'(\vartheta_2) = \frac{3\vartheta_1^* - \vartheta_2 - \vartheta_2 \left(3 \frac{d\vartheta_1^*}{d\vartheta_2} - 1 \right)}{(3\vartheta_1^* - \vartheta_2)^2} = \frac{3(\vartheta_1^* - \vartheta_2 \frac{d\vartheta_1^*}{d\vartheta_2})}{(3\vartheta_1^* - \vartheta_2)^2} \quad (42)$$

In order to determine the sign of the bolded expression in (42) one has to make the following transformations:

$$C = \vartheta_1^* - \vartheta_2 \frac{d\vartheta_1^*}{d\vartheta_2} \quad (43)$$

Now replace $\frac{d\vartheta_1^*}{d\vartheta_2}$ with (34). Using algebraic transformations the following expression is obtained:

$$C = \frac{\vartheta_2(\vartheta_1^* + \vartheta_2)}{9\vartheta_1^{*2} - 6\vartheta_2\vartheta_1^* - \vartheta_2(1 - 2\vartheta_2)} \quad (44)$$

Note that denominator is B (37) which is positive, while numerator is clearly positive:

$$C = \frac{\vartheta_2(\vartheta_1^* + \vartheta_2)}{B} > 0 \quad (45)$$

Going back to the expressions (20) to (23) & (25) it is obvious that as ϑ_2 increases, C is positive causing y_1^{CC} to fall, y_2^{CC} to rise, y^{CC} to fall, p^{CC} to rise and π_2^{CC} to rise.

By multiplying the 2nd part of the expression (24) with $\frac{3\vartheta_1^* - \vartheta_2}{3\vartheta_1^* - \vartheta_2}$ (24) can be rewritten as (46) and apply the known fact from (45):

$$\pi_1^{CC} = \frac{1}{9} - \frac{1}{9} \cdot \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \cdot \frac{3\vartheta_1^* - \vartheta_2}{3\vartheta_1^* - \vartheta_2}$$

$$\pi_1^{CC} = \frac{1}{9} - \frac{1}{9} \cdot \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \left(1 + 2 \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \right) > 0 \quad (46)$$

The same transformation can be applied to (26); By multiplying the 2nd part of the expression with $\frac{3\vartheta_1^* - \vartheta_2}{3\vartheta_1^* - \vartheta_2}$ (26) can be rewritten as (47):

$$\pi^{CC} = \frac{2}{9} + \frac{1}{9} \cdot \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \cdot \frac{3\vartheta_1^* - \vartheta_2}{3\vartheta_1^* - \vartheta_2}$$

$$\pi^{CC} = \frac{2}{9} + \frac{1}{9} \cdot \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \left(1 - \frac{\vartheta_2}{3\vartheta_1^* - \vartheta_2} \right) \quad (47)$$

It is necessary to determine dynamics of the second part of the expression, which can be rewritten as $D(1 - D)$ when (41) is applied. Graph of this equation is a parabola which increases for $D < 0.5$. However, due to the fact that $\vartheta_1^* > \vartheta_2$ (30), D always fulfills that requirement, concluding that (47) increases as ϑ_2 increases.

These findings bring to conclusion that as the conflict of interest increases, measured with the Agent's share in Company 2, the market shows tendencies towards monopoly: the overall quantity produced diminishes, prices soar as well as the Company 2 and the total market profit, while Company 1 gradually declines and disappears when ϑ_2 reaches 0.5.

3.3. Numeric analysis

The following table contains numeric simulation of the effects of different Agent's shares in Company 2 on the overall market quantities, profits and price.

Table 2 Simulation of the effects of different Agent's shares in Company 2

ϑ_2	0.05	0.25	0.50	0.75	Basic Cournot model
ϑ_1^*	0,168	0,447	0,68	0,859	
y_1^{CC}	0,26	0,181	0,117	0,06	0,333
y_2^{CC}	0,37	0,41	0,442	0,47	0,333
y^{CC}	0,63	0,59	0,558	0,53	0,667
p^{CC}	0,37	0,41	0,442	0,47	0,333
π_1^{CC}	0,096	0,074	0,052	0,028	0,111
π_2^{CC}	0,137	0,168	0,195	0,221	0,111
π^{CC}	0,233	0,242	0,247	0,249	0,222

Source: the authors

ϑ_2 values are randomly picked, ϑ_1^* is solved by solving (32) for the given ϑ_2 and the rest of the

values are obtained by stubbing the ϑ_1^* and ϑ_2 into (20) – (26). Z function for the scenarios given in the Table 2, as well as the $\vartheta_2 = 0$ scenario (linear Z function), is given in the Figure 3.

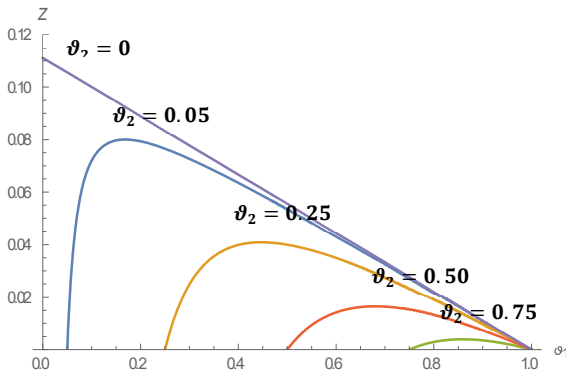


Figure 3 Principal's layer curves of the profit function Z for different values of ϑ_2
Source: the authors

Figure 3 shows that the larger the Agent's share in the competitive company, the greater the ϑ_1 share that the Principal has to give to the Agent, and the maximum payoff of the owner (Z) depletes gradually as ϑ_2 increases. Note that Figure 3 maps vertical cuts of the Z function shown on Figure 2.

Conclusion

This paper analyzes a basic Cournot duopoly model with standardized prices and zero costs. It was proposed that the agent of company 1 (Agent) is rewarded with a share of profits by the company's owner (Principal). A share of the other company is owned by the Agent, but without executive power. The purpose of this study was to demonstrate how the conflict of interests increases as the Agent's share of the other company increases. Principal's profit function is positive only when he rewards the Agent with a greater share than the Agent's share in the other company, but depletes as that share increases. Using comparative statics, it is shown that these two shares are positively correlated. Also, it has been shown that as the Agent's share in the competitive company increases, the market gradually becomes monopolized, as the overall quantity produced decreases. As the market price increases, the profits of both the competitive company and the agent also increase, while the company they run is gradually choked by the Agent, depleting its profits and reducing its production as a result. Consequently, the greater the conflict of interest, the lower the consumer's welfare and the lower the level of competitiveness.

The paper presents a novel method of measuring competitiveness by relating the market coverage to the equivalent number of companies in an oligopoly, demonstrating that the conflict of interests is the same as if the number of companies were not integer. Future studies will focus on generalizing the analysis to more participants, not only two.

Declarations

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

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